



## Remarques

[ 4 remarque(s) ]

- Cours très intéressant et bien enseigné !
- Cours très intéressant, avec un professeur qui donne vraiment envie de comprendre et travailler la matière. L'ambiance lors du cours est vraiment très sympathique, et la correction des exercices, puis le cours est un très bon format. C'est toujours un plaisir de venir à ce cours !
- The course is well structured and Prof. Besse is obviously very enthusiastic to giving the lectures. Its relevance, purpose and organisation are very well defined. The intuition-based approach is also very refreshing. Thank you very much, Prof. Besse!
- Tout bon ! Pas besoin de changement jusqu'à maintenant

**La science quantique  
Une vision singulière**

**VI) Densités  
et statistiques**

P.A. Besse

## Combien de pain pour nourrir cet immeuble ?



La Grande Motte

$\rho(E)$  = nombre d'appartements à chaque étage E

$F(E)$  = taux de personnes en moyenne  
par appartement

$c(E)$  = consommation moyenne par personne



$$= \sum_E c(E) \cdot \rho(E) \cdot F(E)$$

## Deux exemples d'utilisation en physique

### Concentration d'électrons libres dans un semi-conducteur:

$$n = \int_{E_c}^{\infty} (2) \cdot \rho_e^{3D}(E) \cdot F_e(E) \cdot dE$$

Contribution d'un état      Densité d'états      Taux d'occupation

$$\frac{P_{tot}}{S \cdot \Omega_{\max}} = \int_0^{\infty} \left( 2 \frac{c}{4\pi} E \right) \cdot \rho_{\gamma}^{3D}(E) \cdot F_{\gamma}(E) \cdot dE$$

### Puissance totale émise par un corps noir par surface et angle d'émission:

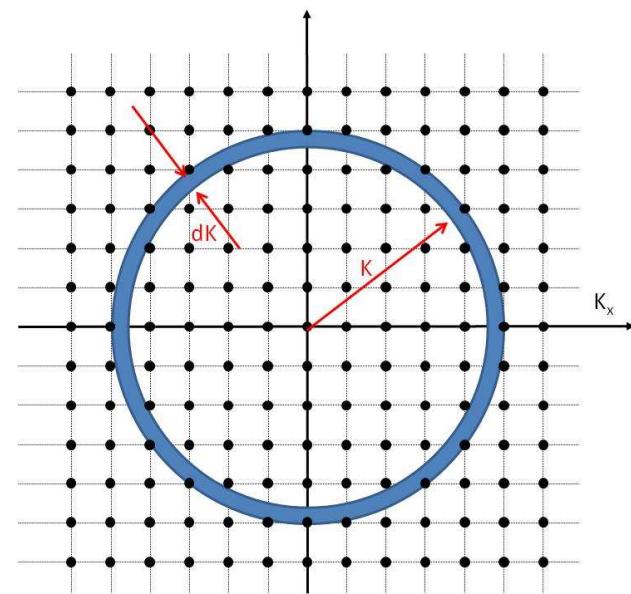
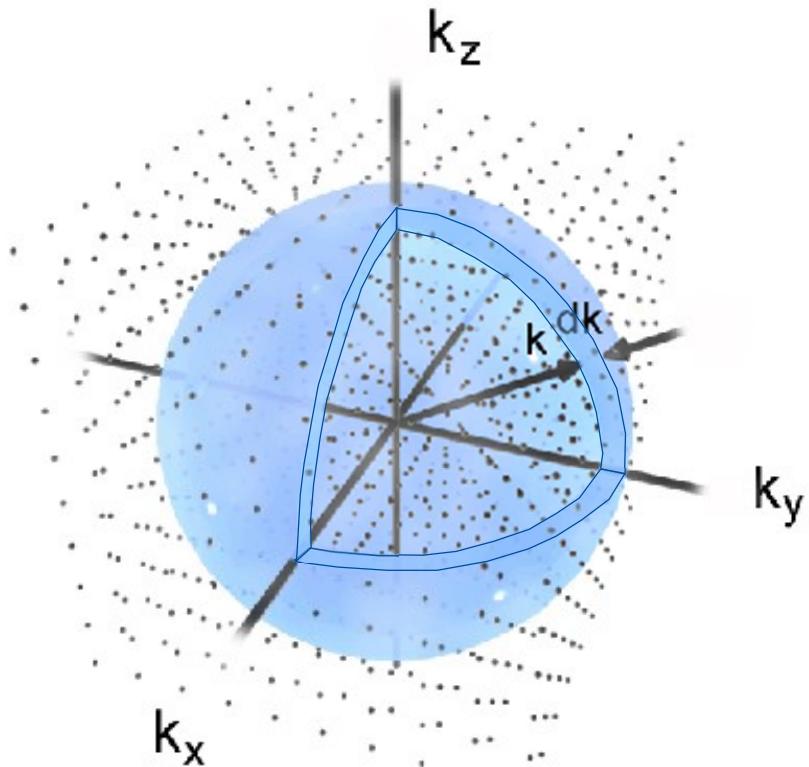
# Densités d'états

**3D**

# Densité d'états de même norme de K

$$n \cdot \lambda = L \Rightarrow K = \frac{2\pi}{\lambda} = n \cdot \frac{2\pi}{L}$$

→ Réseau régulier  $\Delta K = \frac{2\pi}{L}$



$$n_K = (4\pi \cdot K^2 \cdot dK) / \Delta K^3$$

La densité d'états avec la même norme de K:

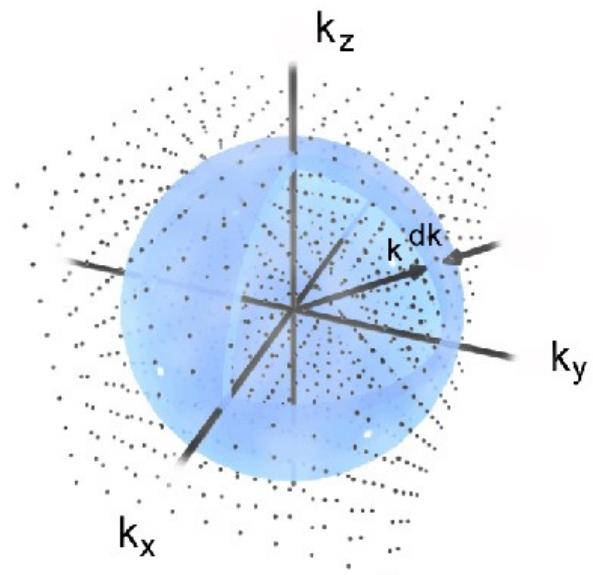
$$\rho_K \cdot dK \equiv \frac{n_K}{L^3} = \left( \frac{1}{2\pi} \right)^3 \cdot 4\pi \cdot K^2 \cdot dK$$

# Densité d'états pour les électrons

La densité d'états avec la même norme de K:

$$\rho_K \cdot dK \equiv \frac{n_K}{L^3} = \left( \frac{1}{2\pi} \right)^3 \cdot 4\pi \cdot K^2 \cdot dK$$

Dispersion:



Densité d'état en énergie (Electrons):

$$E_e = \frac{p^2}{2m^*} = \frac{\hbar^2}{2m^*} \cdot K^2$$



$$\sqrt{E} \approx K \quad dE \approx K \, dK$$

$$K^2 \cdot dK = K \cdot K \, dK \approx \sqrt{E} \cdot dE$$

$$\rho_E \cdot dE = \frac{1}{\sqrt{2}} \frac{m^{*3/2}}{\pi^2 \hbar^3} \cdot \sqrt{E - E_0} \cdot dE$$

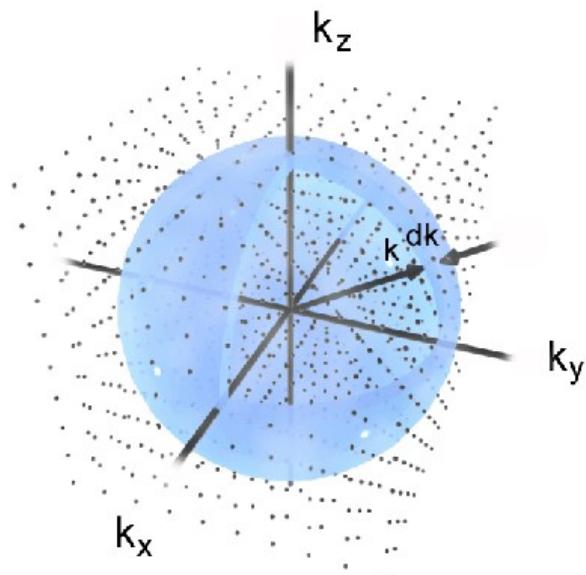
$$\left[ \frac{1}{m^3} \right]$$

# Densité d'états pour les photons

La densité d'états avec la même norme de K:

$$\rho_K \cdot dK \equiv \frac{n_K}{L^3} = \left( \frac{1}{2\pi} \right)^3 \cdot 4\pi \cdot K^2 \cdot dK$$

Dispersion:



$$E_\gamma = c\hbar \cdot K$$



$$dE \approx c \cdot \hbar \cdot dK$$

$$K^2 \cdot dK = E^2 / (c\hbar)^3 \cdot dE$$

Densité d'état en énergie (photons):

$$\rho_E \cdot dE = \frac{1}{2\pi^2} \cdot \frac{E^2}{c^3 \hbar^3} \cdot dE \quad \left[ \frac{1}{m^3} \right]$$

# Résumé des densités 3D

$$\rho_K^{3D} \cdot dK = \left( \frac{1}{2\pi} \right)^3 \cdot 4\pi \cdot K^2 \cdot dK$$

## Electrons

$$E_e = \frac{\hbar^2}{2m^*} \cdot K^2$$

$$\rho_{E,e}^{3D} \cdot dE = \frac{1}{\sqrt{2 \cdot \pi^2}} \cdot \frac{m^{*3/2}}{\hbar^3} \cdot \sqrt{E - E_0} \cdot dE \quad \left[ \frac{1}{m^3} \right]$$

## Photons

$$E_\gamma = c \cdot \hbar K$$

$$\rho_{E,\gamma}^{3D} \cdot dE = \frac{1}{2\pi^2} \cdot \frac{E^2}{c^3 \hbar^3} \cdot dE \quad \left[ \frac{1}{m^3} \right]$$

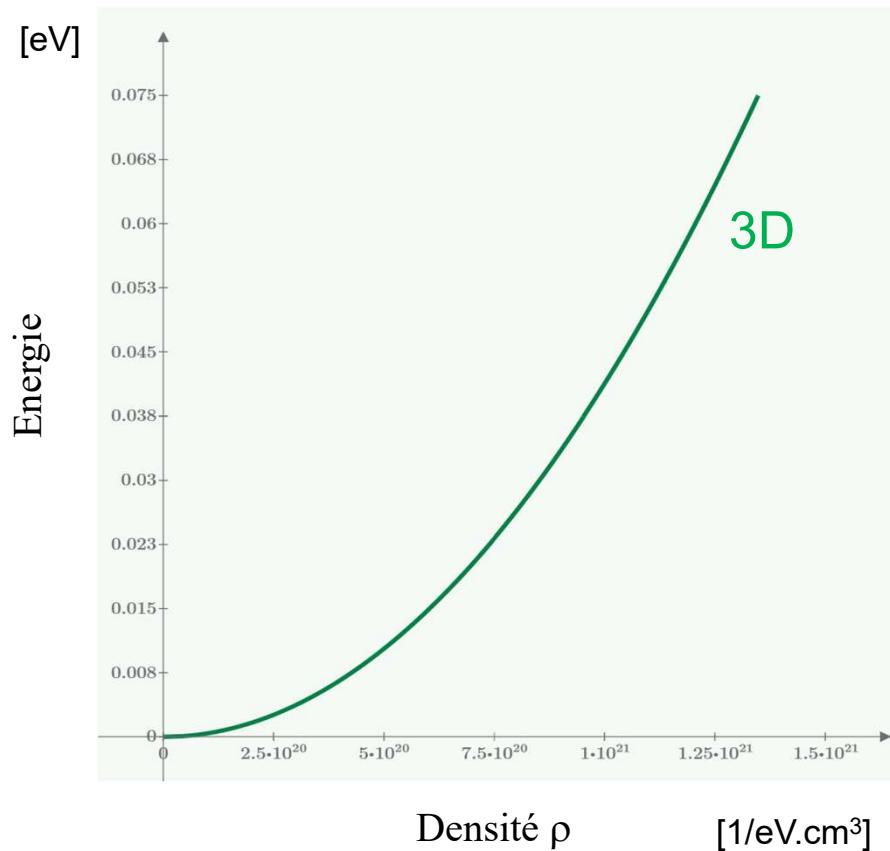
$$E = \hbar \cdot 2\pi\nu \Rightarrow \rho_{\nu,\gamma}^{3D} \cdot d\nu = \frac{4\pi}{c^3} \cdot \nu^2 \cdot d\nu$$

$$\nu = c/\lambda \Rightarrow \rho_{\lambda,\gamma}^{3D} \cdot d\lambda = 4\pi \frac{1}{\lambda^4} \cdot d\lambda$$

# Exemple

## Electrons

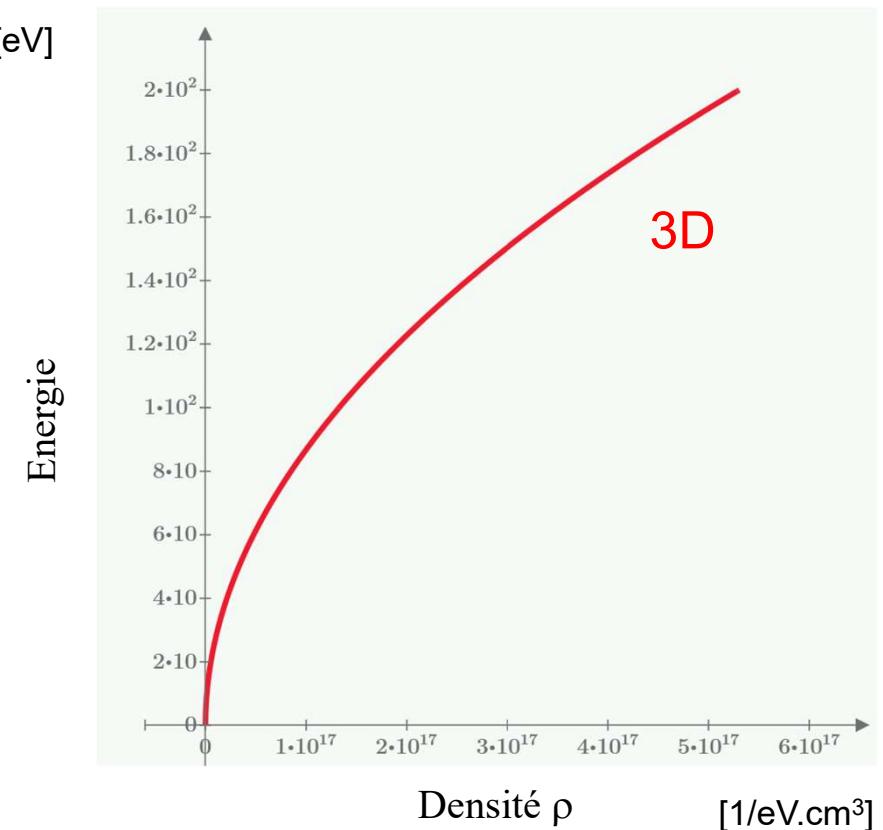
$$\rho_{E,e}^{3D} \cdot dE = \frac{1}{\sqrt{2} \cdot \pi^2} \cdot \frac{m^{*3/2}}{\hbar^3} \cdot \sqrt{E - E_0} \cdot dE$$



$$m_e/m_0 = 0.8$$

## Photons

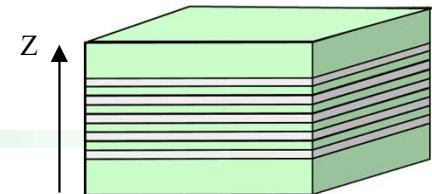
$$\rho_{E,\gamma}^{3D} \cdot dE = \frac{1}{2\pi^2} \cdot \frac{E^2}{c^3 \hbar^3} \cdot dE$$



# Densité d'états électroniques

## 2D et 1D

# Quantum Wells (2D)



$$n \cdot \frac{\lambda}{2} = L_z$$

$n=1,2,3,\dots$

$$\Downarrow$$

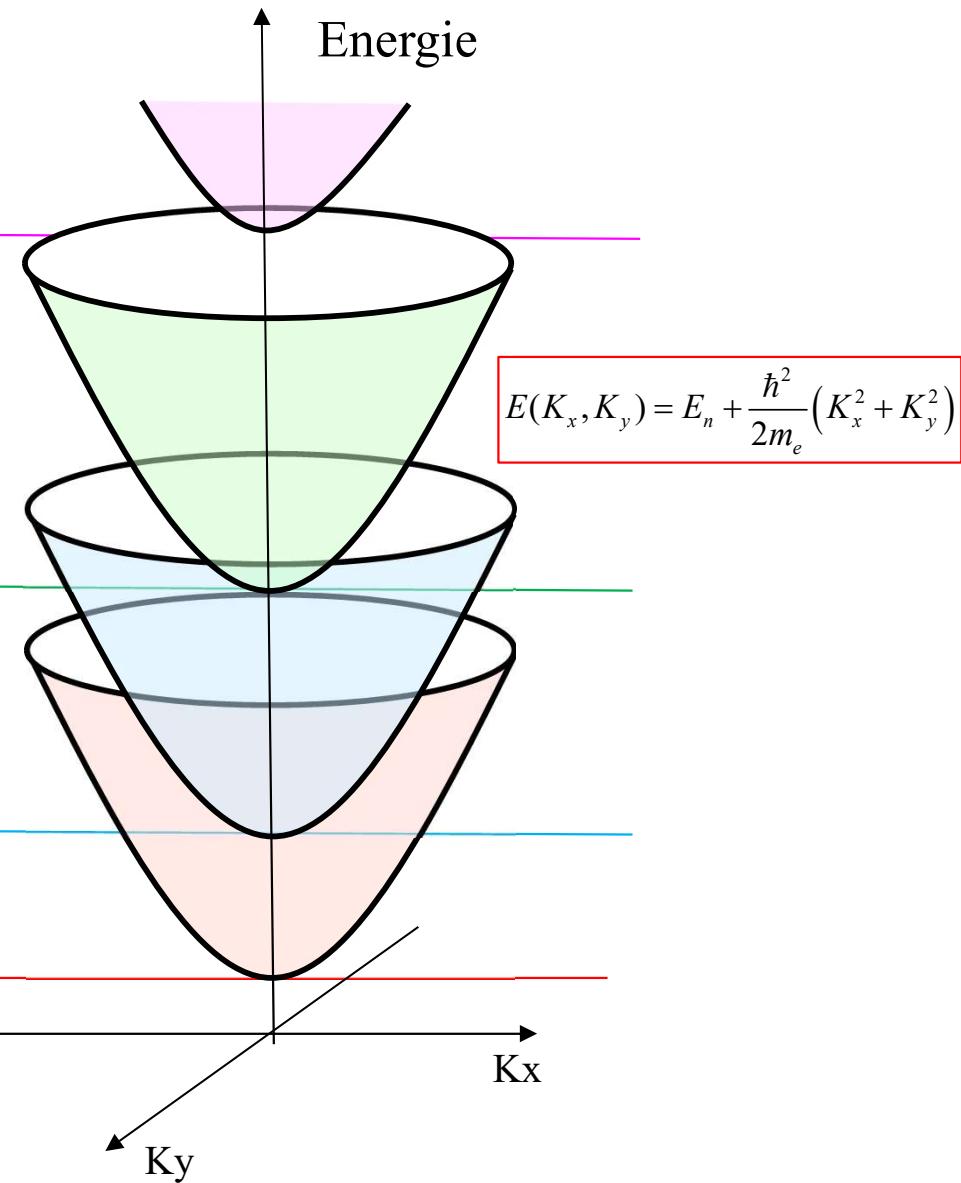
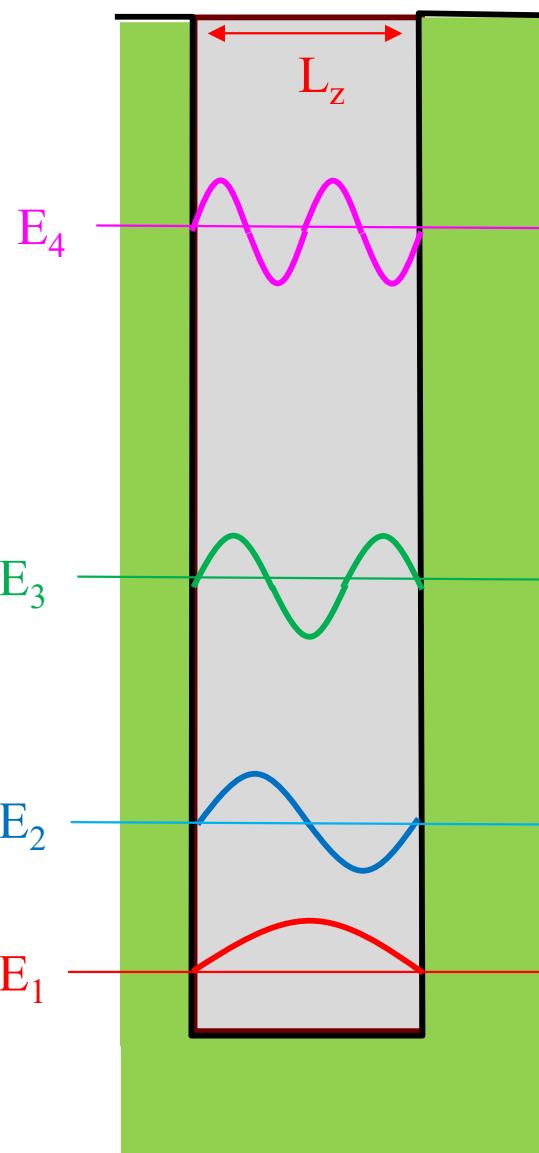
$$K_z = n \cdot \frac{\pi}{L_z}$$

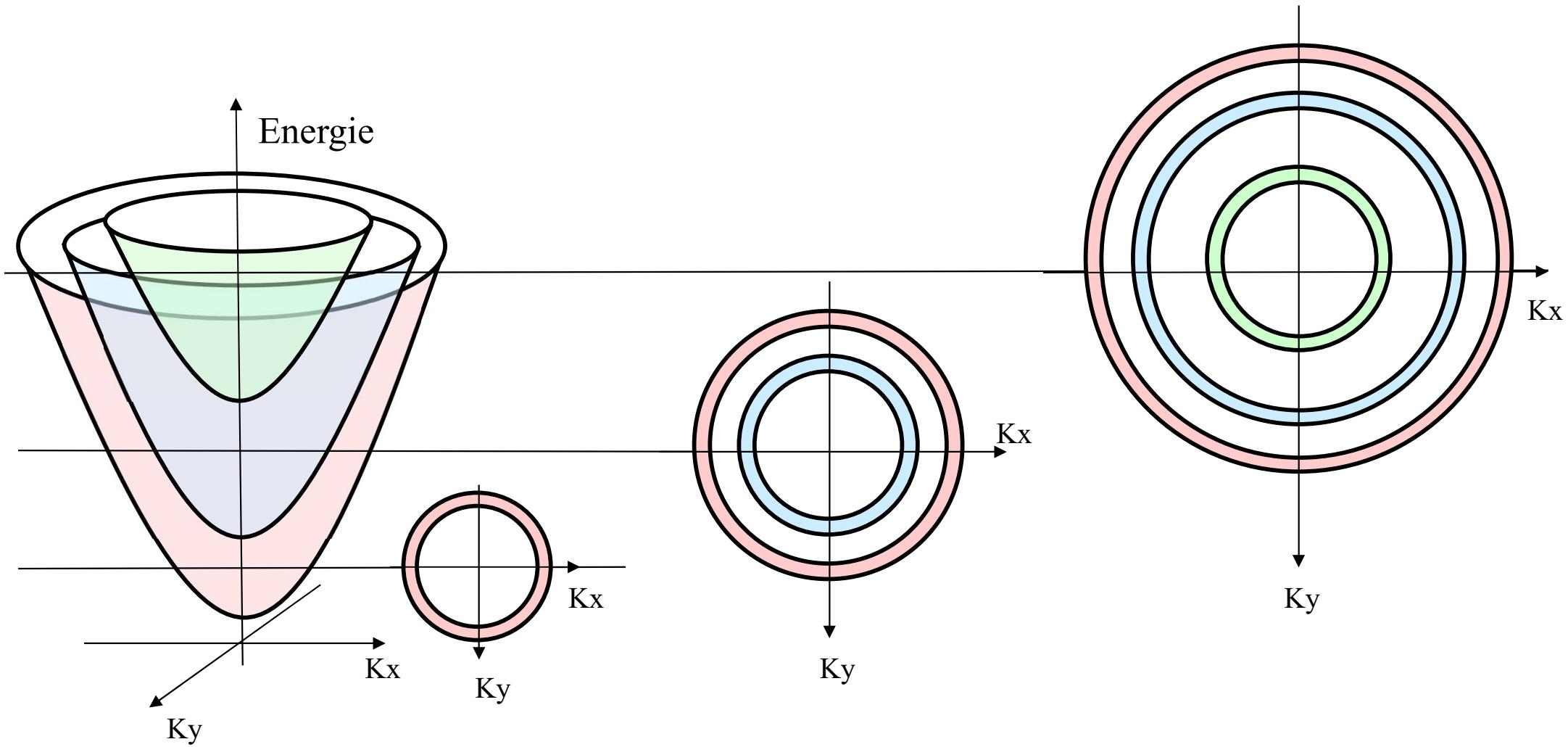
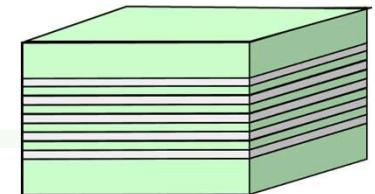
$\Downarrow$

$$E_n = \frac{\hbar^2 K_z^2}{2m_e}$$

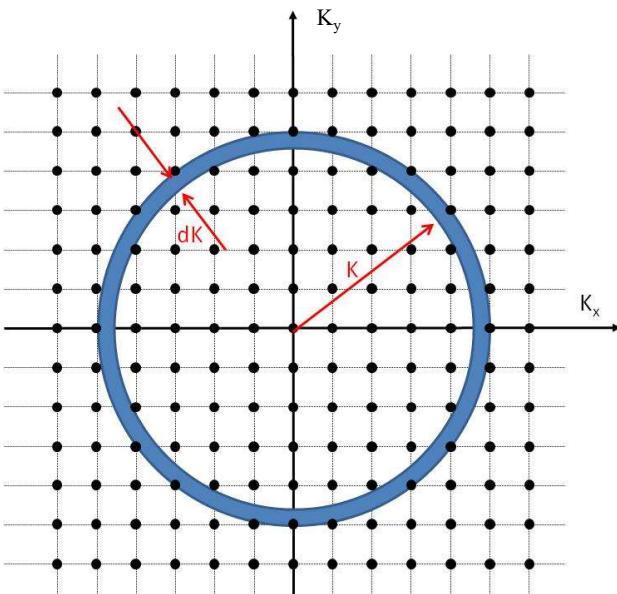
$\Downarrow$

$$E_n = n^2 \cdot \frac{\hbar^2}{2m_e} \frac{\pi^2}{L_z^2}$$





# Densité électronique 2D



$$K = \frac{2\pi}{L}$$

$$E = E_n + \frac{\hbar^2}{2m_e} K^2$$



$$dE = \frac{\hbar^2}{m^*} \cdot K \cdot dK$$



$$n_K = (2\pi \cdot K \cdot dK) / \Delta K^2$$

La densité d'états avec la même norme de K:

$$\rho_K \cdot dK \equiv \frac{n_K}{L^2} = \frac{1}{2\pi} \cdot K \cdot dK$$

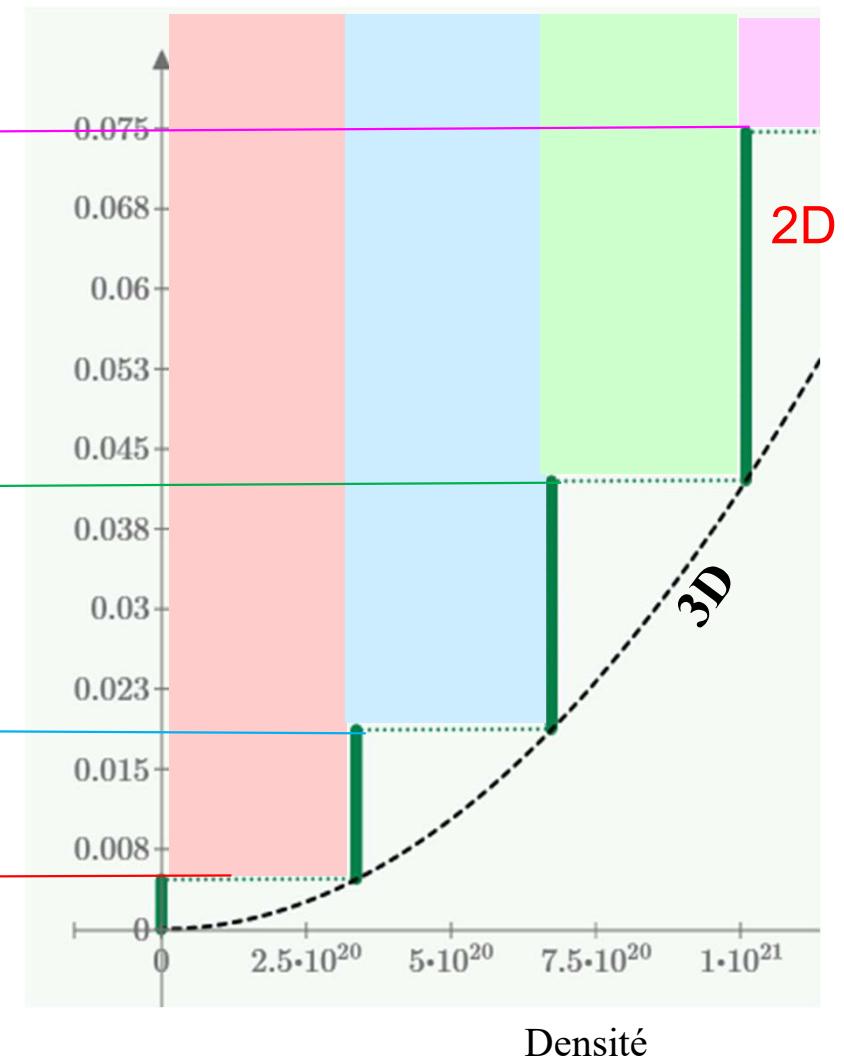
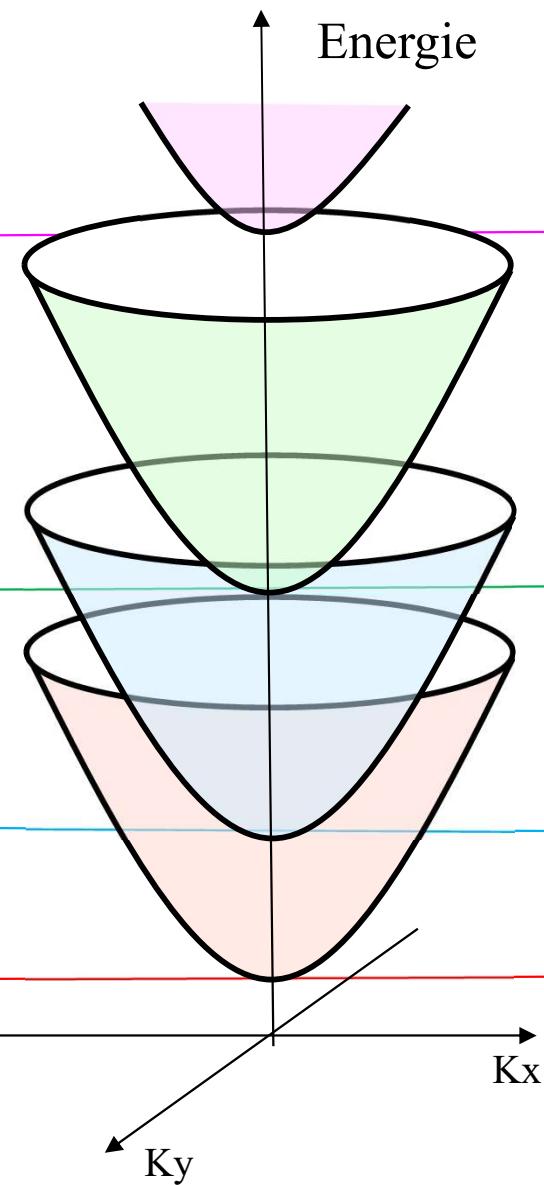
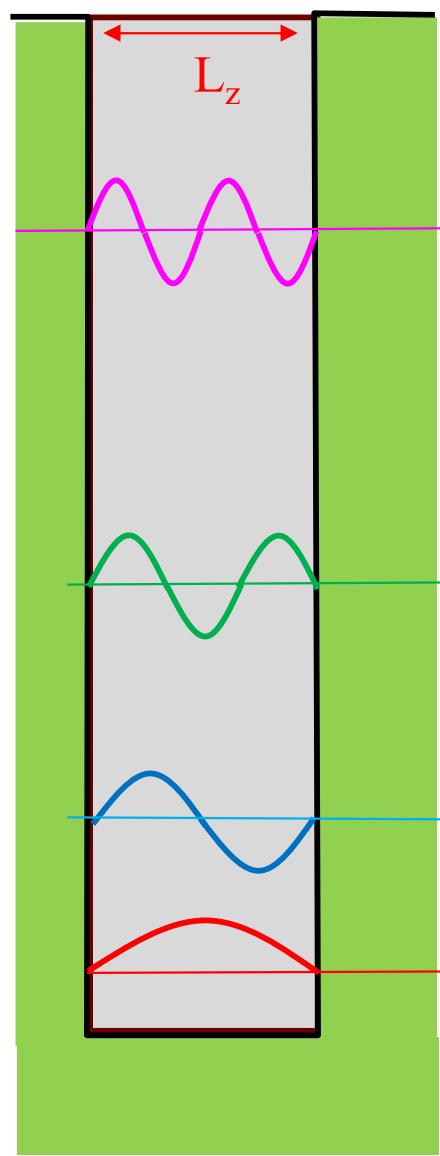
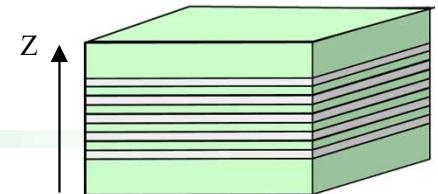


si  $E \geq E_n$

$$\rho_{E,e}^{2D} \cdot dE = \frac{1}{2\pi} \cdot \frac{m^*}{\hbar^2} \cdot dE$$

$$\left[ \frac{1}{m^2} \right]$$

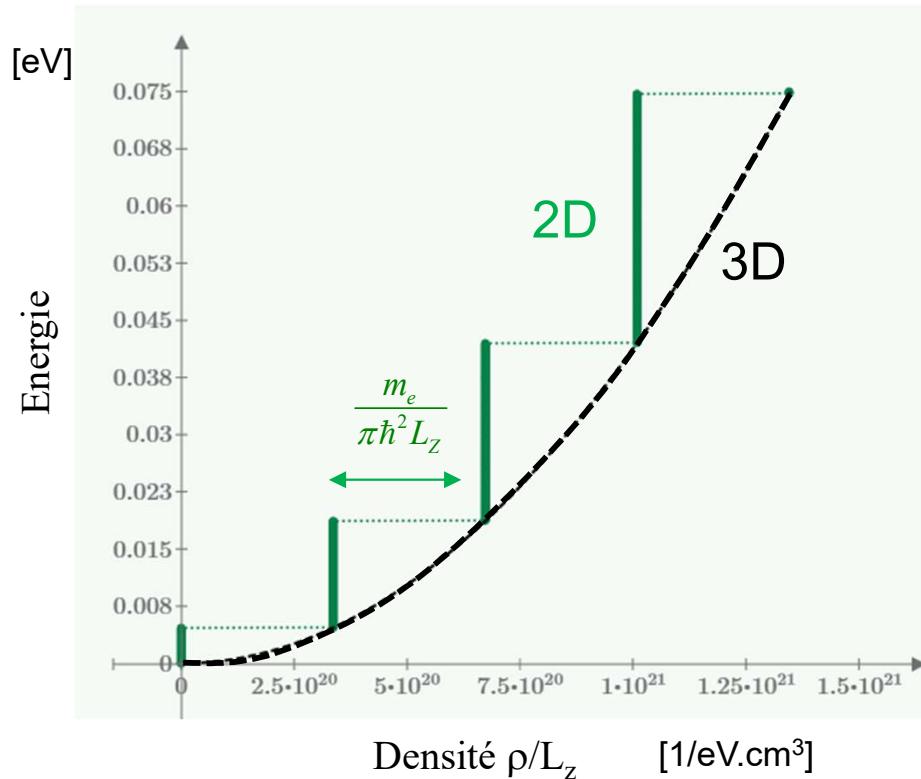
# Quantum Wells (2D)



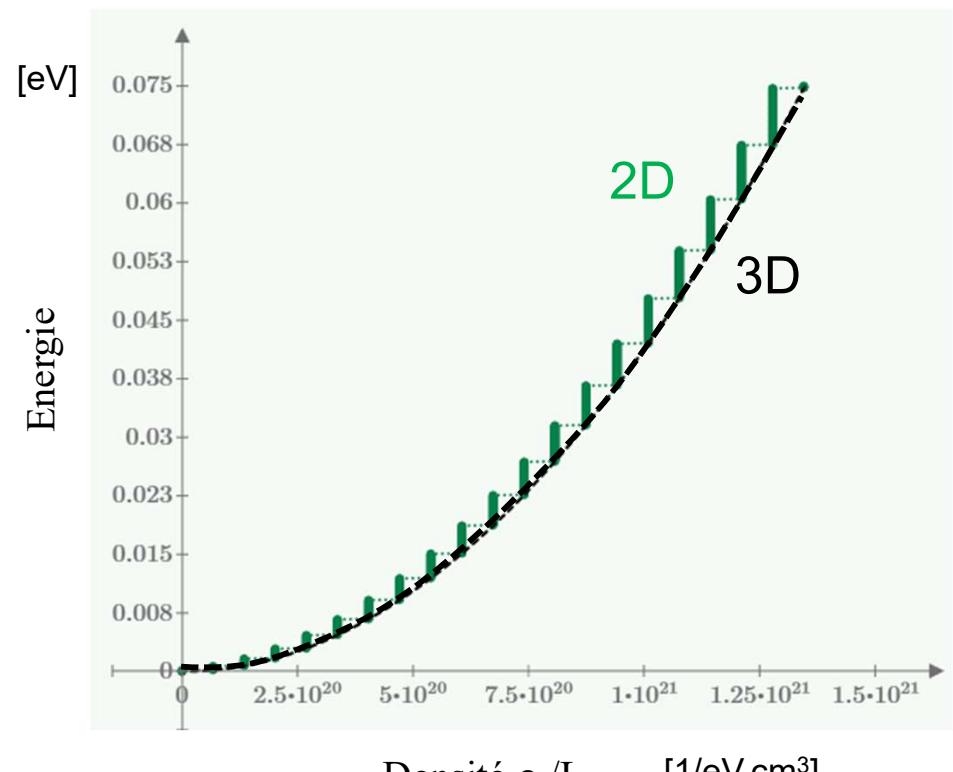
# Densité d'états électroniques 2D

$$\frac{\rho_{tot}^{2D}(E)}{L_z} = 2 \cdot \frac{1}{2\pi} \cdot \frac{1}{L_z} \cdot \frac{m_e}{\hbar^2} \cdot \sum_{n=1}^{\infty} H(E - E_n)$$

$$m_e/m_0=0.8$$

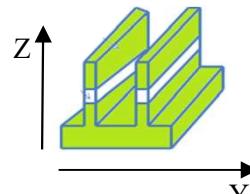


$$L_z = 10 \text{ nm}$$



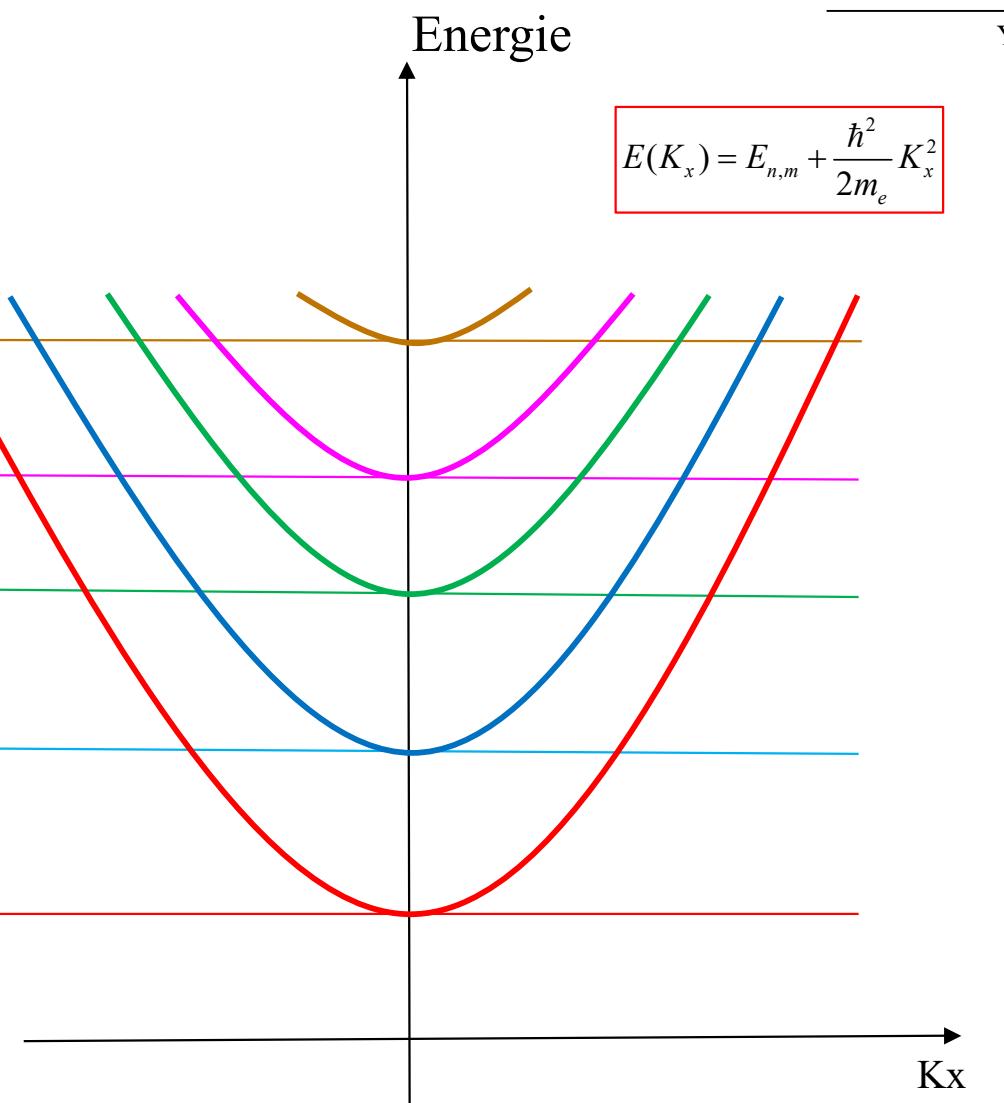
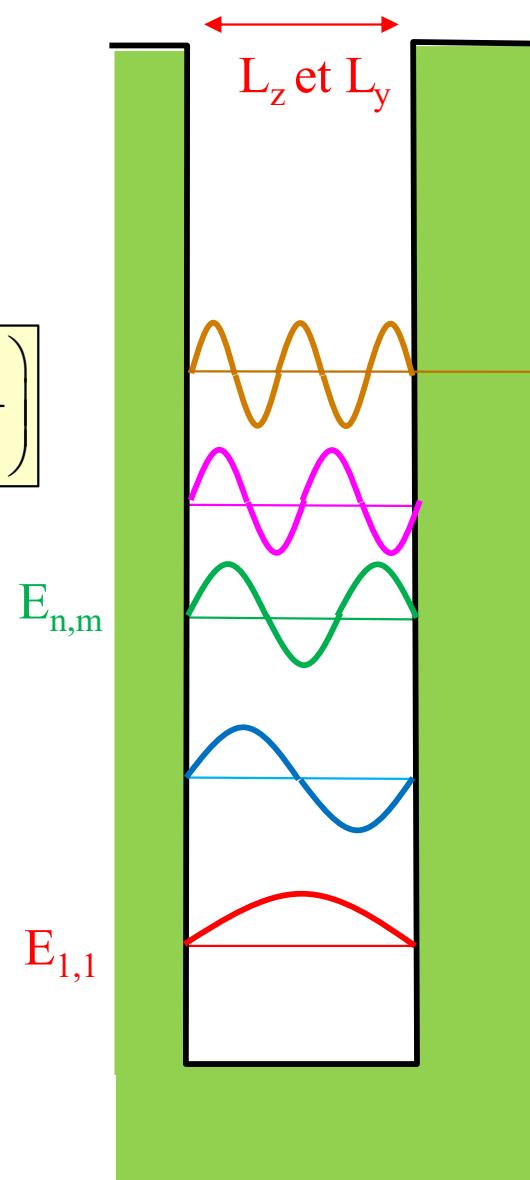
$$L_z = 50 \text{ nm}$$

# Quantum Wires (1D)

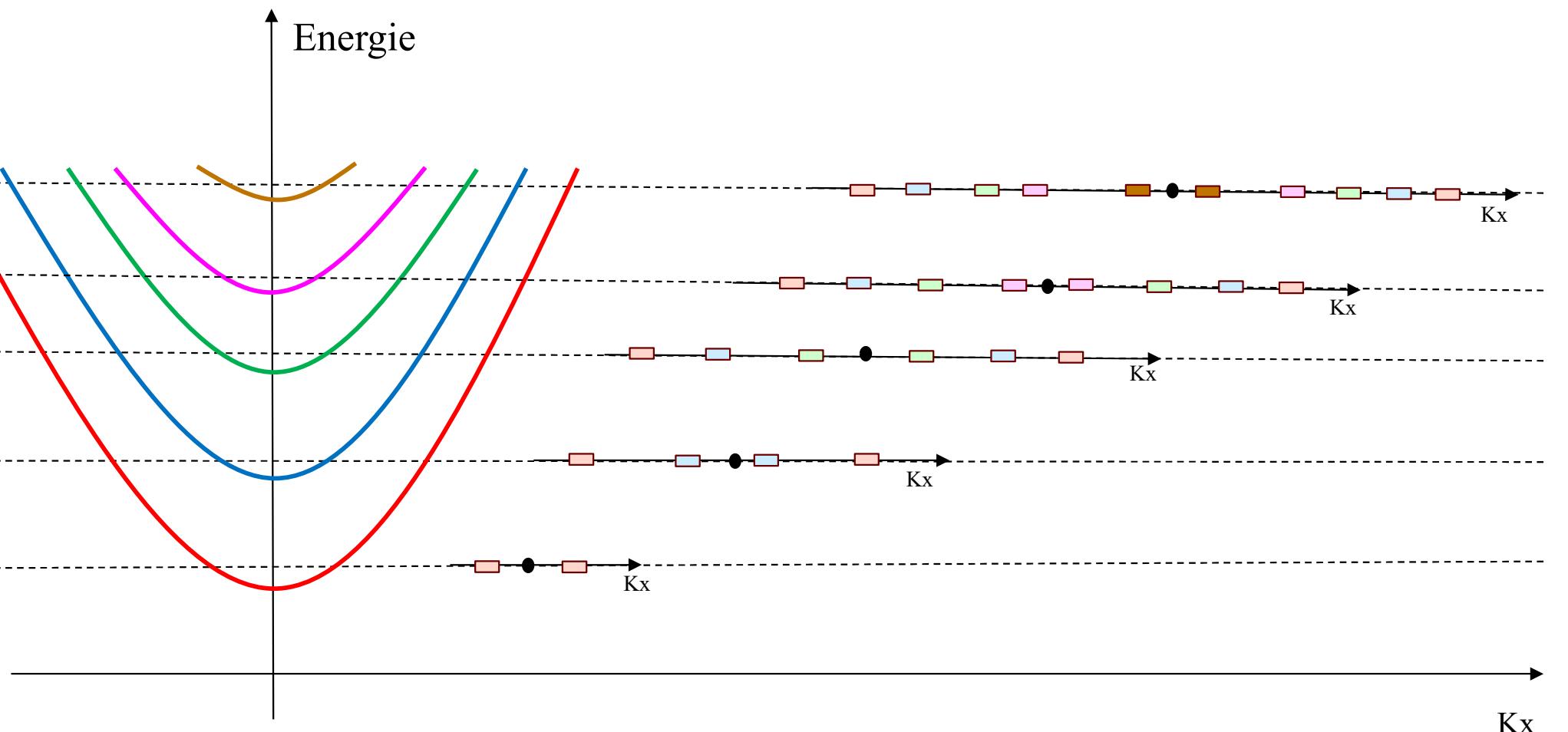
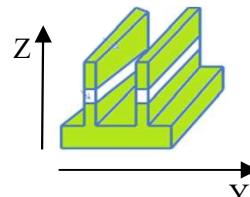


$$E_{n,m} = \frac{\hbar^2}{2m_e} \pi^2 \left( \frac{n^2}{L_z^2} + \frac{m^2}{L_y^2} \right)$$

$n=1,2,3,\dots$   
 $m=1,2,3,\dots$

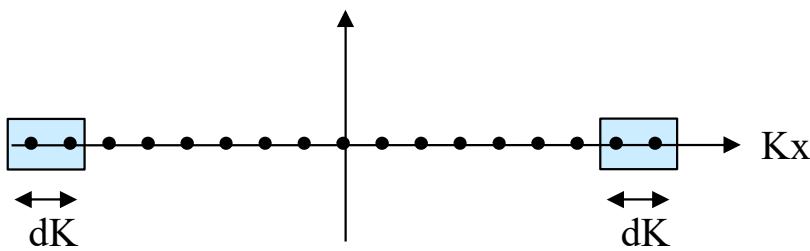


# Quantum Wires (1D)



# Densité électronique 1D

$$\Delta K = \frac{2\pi}{L}$$



$$n_K = 2 \cdot dK / \Delta K$$

La densité d'états avec la même norme de K:

$$\rho_K \cdot dK \equiv \frac{n_K}{L} = \frac{1}{\pi} \cdot dK$$



$$E = E_{n,m} + \frac{\hbar^2}{2m_e} K^2$$



$$dE = \frac{\hbar^2}{m^*} K \cdot dK = \sqrt{\frac{2\hbar^2}{m^*}} \sqrt{E - E_{n,m}} \cdot dK$$

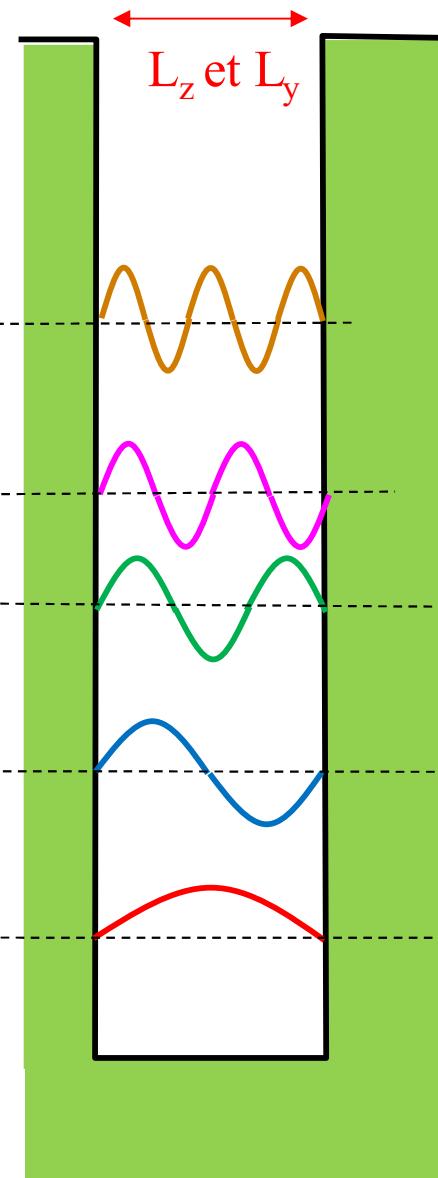
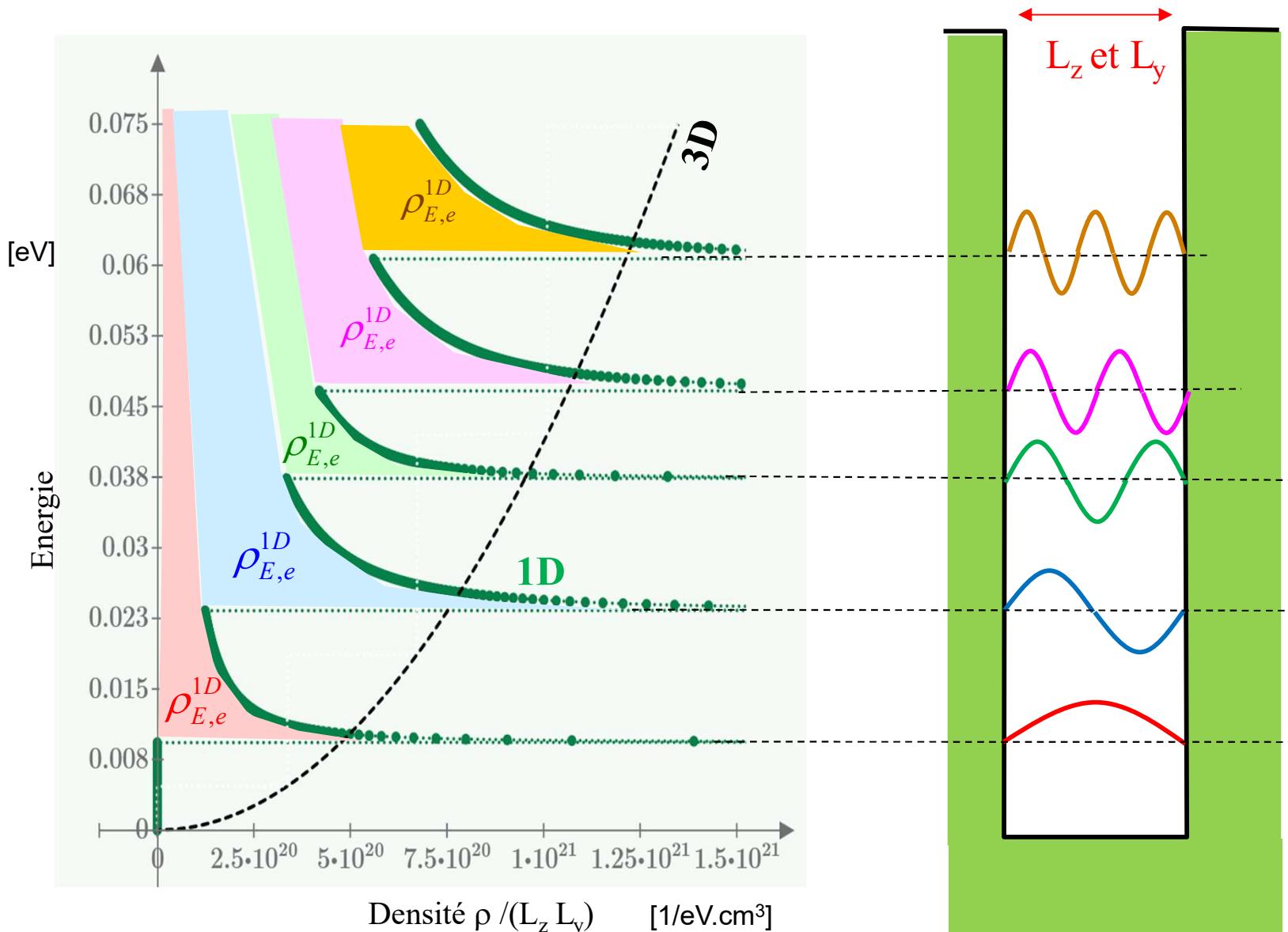


$$\text{si } E \geq E_{n,m}$$

$$\rho_{E,e}^{1D} \cdot dE = \frac{1}{\pi} \cdot \sqrt{\frac{m^*}{2\hbar^2}} \frac{1}{\sqrt{E - E_{n,m}}} \cdot dE$$

$$\left[ \frac{1}{m} \right]$$

# Densité électronique 1D



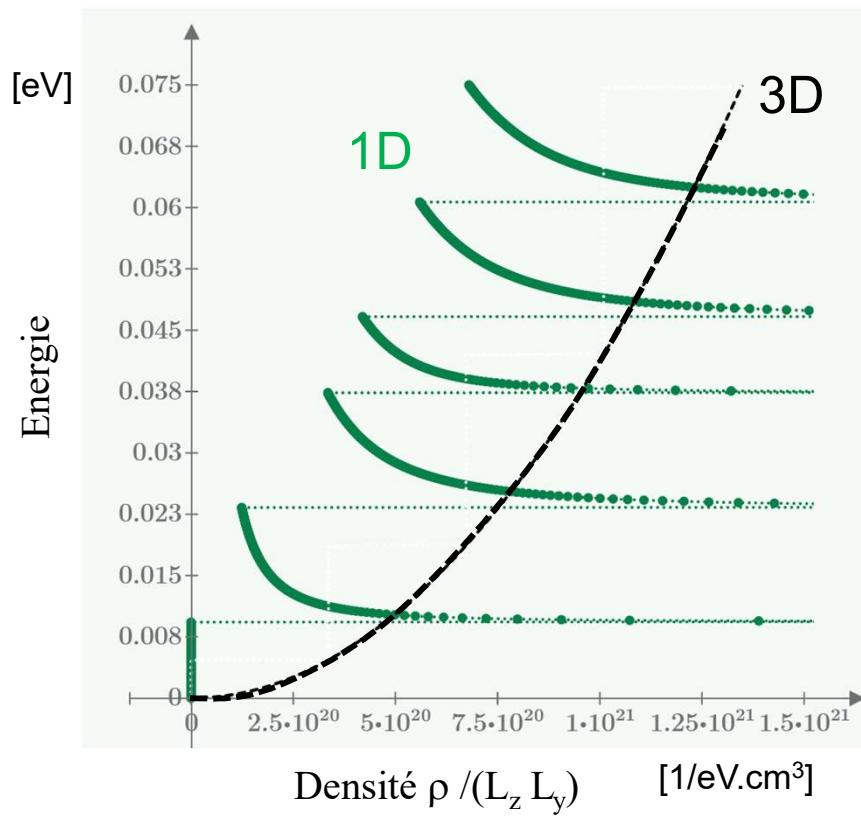
$$E_{n,m} = \frac{\hbar^2}{2m_e} \pi^2 \left( \frac{n^2}{L_z^2} + \frac{m^2}{L_y^2} \right)$$

$n=1,2,3,\dots$   
 $m=1,2,3,\dots$

# Densité d'états électroniques 1D

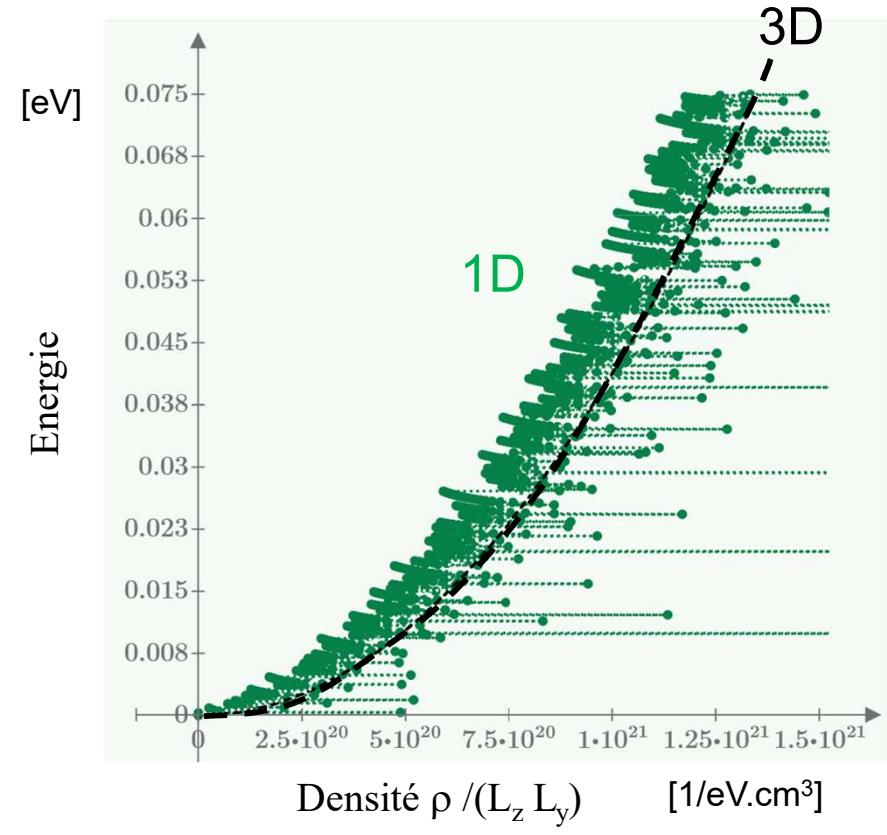
Densité volumique d'états totale:

$$\frac{\rho_{tot}^{1D}(E)}{L_y L_z} = 2 \cdot \frac{1}{\pi} \cdot \frac{\sqrt{m_e}}{\sqrt{2\hbar^2}} \cdot \frac{1}{L_y L_z} \cdot \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left( \frac{H(E - E_{n,m})}{\sqrt{E - E_{n,m}}} \right)$$



$$L_z = L_y = 10 \text{ nm}$$

$$m_e/m_0 = 0.8$$

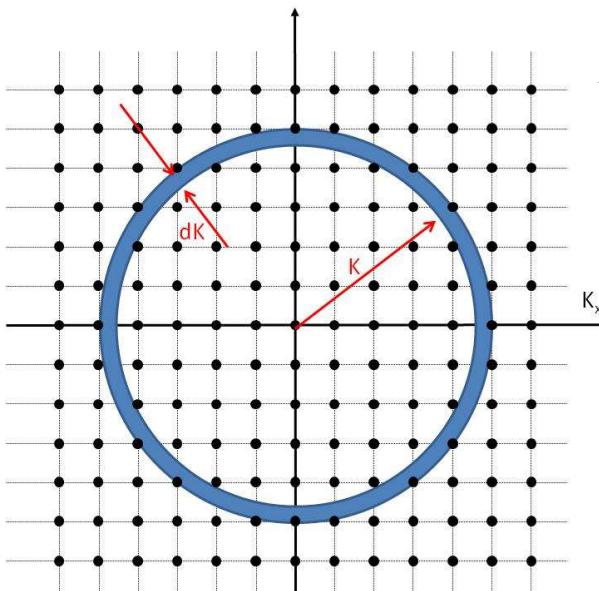


$$L_z = L_y = 50 \text{ nm}$$

# Densité d'états photoniques

## 2D et 1D

# Densité photonique 2D

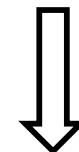


$$\Delta K = \frac{2\pi}{L}$$

$$E^2 = E_n^2 + (c \cdot \hbar K)^2$$



$$K \cdot dK = \frac{1}{c^2 \hbar^2} E \cdot dE$$

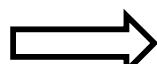


$$n_K = (2\pi \cdot K \cdot dK) / \Delta K^2$$

La densité d'états avec la même norme de K:

$$\text{si } E \geq E_n$$

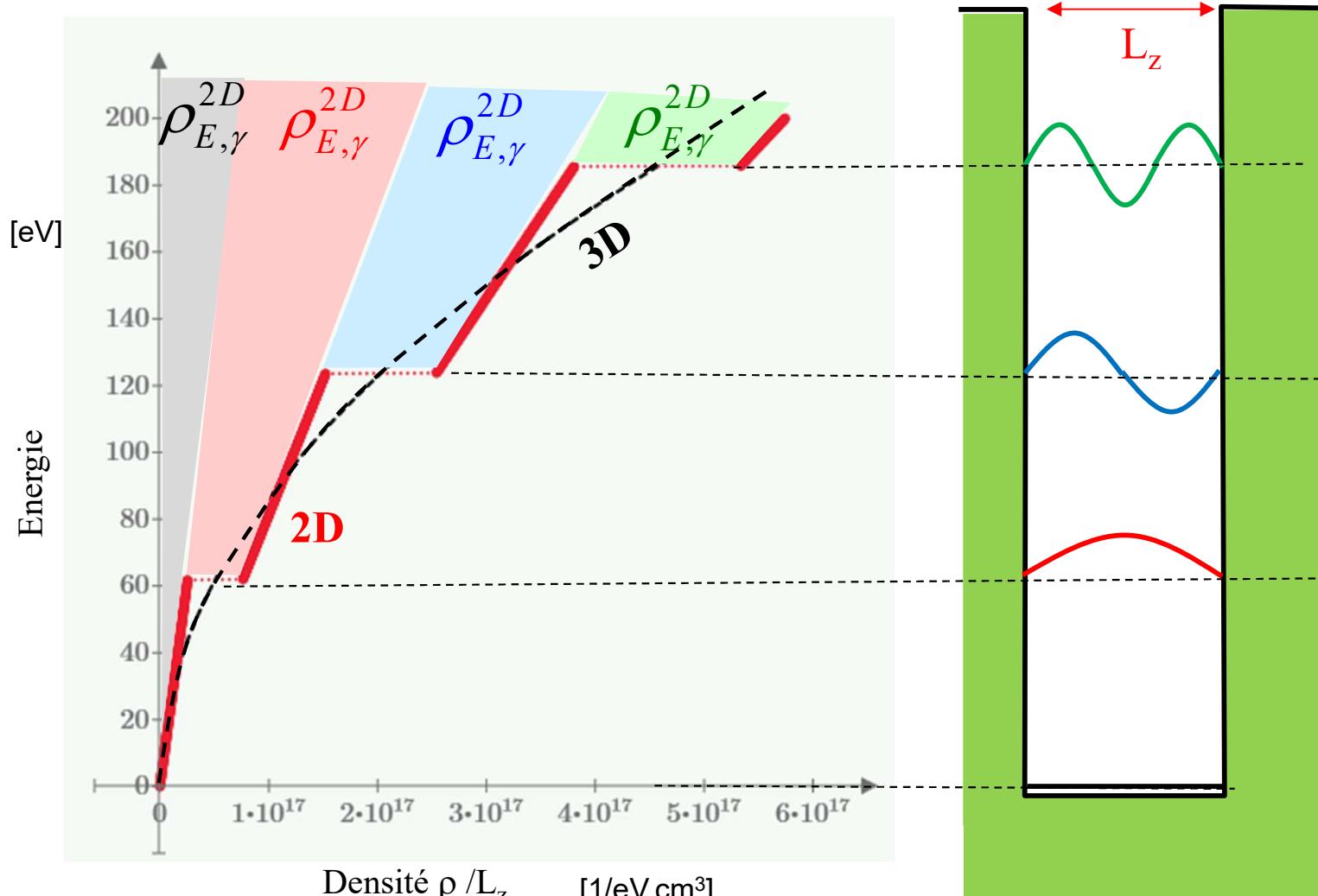
$$\rho_K \cdot dK \equiv \frac{n_K}{L^2} = \frac{1}{2\pi} \cdot K \cdot dK$$



$$\rho_{E,\gamma}^{2D} \cdot dE = \frac{1}{2\pi} \cdot \frac{E}{c^2 \hbar^2} \cdot dE$$

$$\left[ \frac{1}{m^2} \right]$$

# Densité photonique 2D



$$n \cdot \frac{\lambda}{2} = L_z$$

$n=1,2,3,\dots$



$$K_z = n \cdot \frac{\pi}{L_z}$$



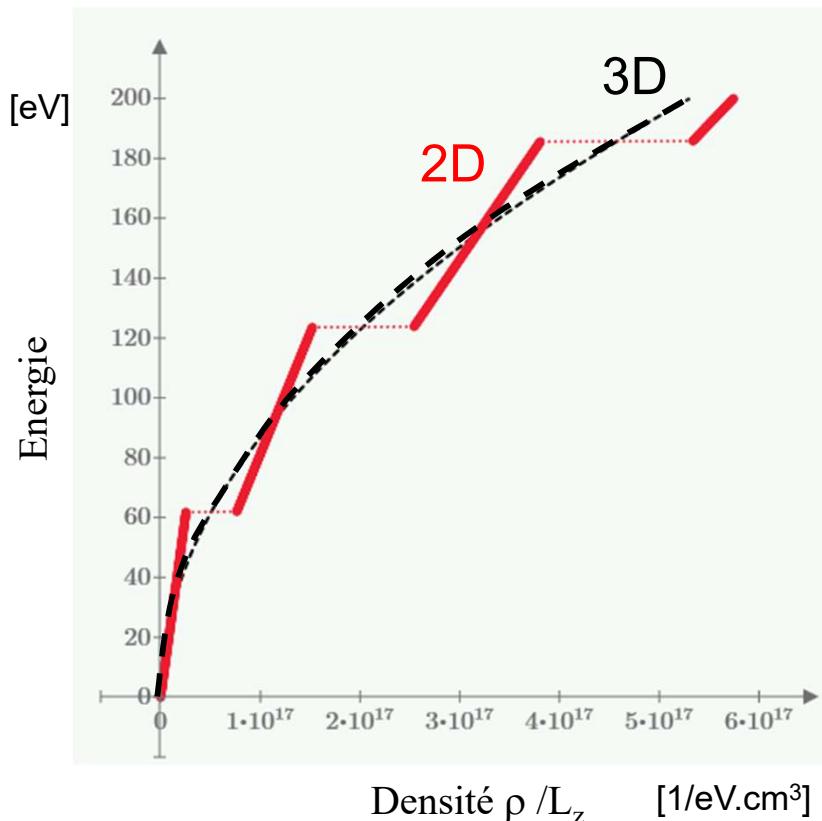
$$E_n = \hbar c \cdot |K_z|$$



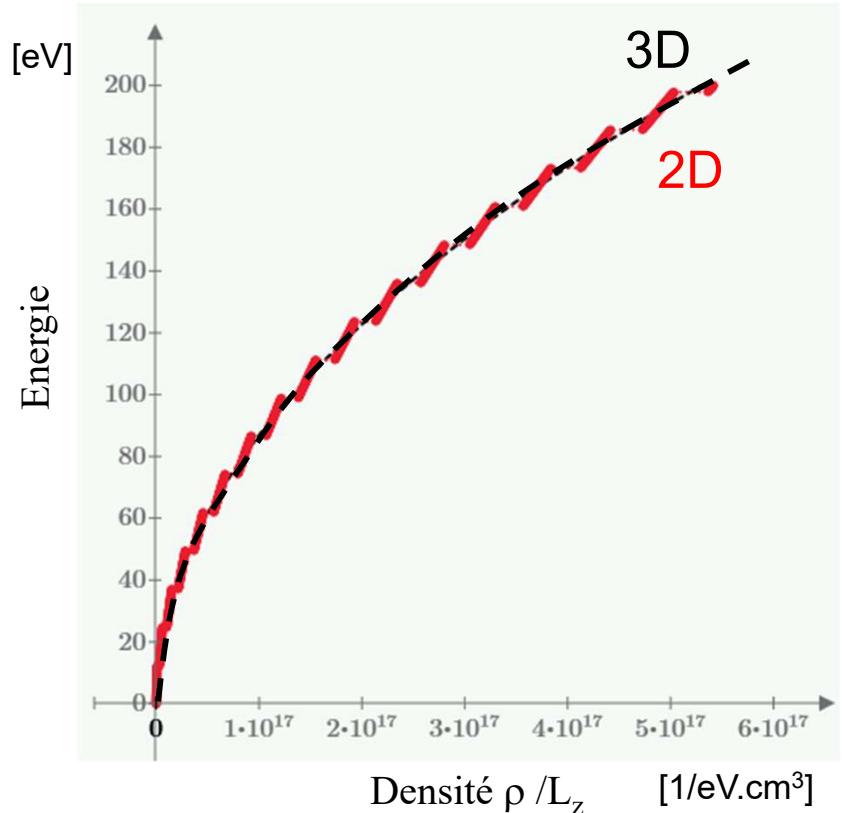
$$E_n = n \cdot \hbar c \frac{\pi}{L_z}$$

# Densité d'états photoniques 2D

$$\frac{\rho_{tot}^{2D}(E)}{L_z} = \frac{1}{2\pi\hbar^2c^2} \frac{1}{L_z} \cdot \left[ \textcolor{red}{1} \cdot E \cdot H(E - E_0) + \textcolor{red}{2} \sum_{n=1}^{\infty} E \cdot H(E - E_n) \right]$$



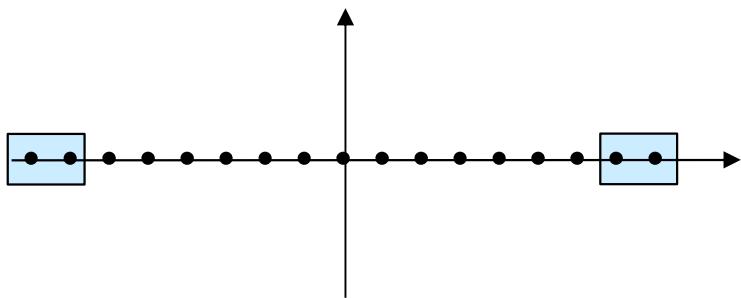
$L_z=10\text{nm}$



$L_z=50\text{nm}$

# Densité photonique 1D

$$\Delta K = \frac{2\pi}{L}$$



$$E^2 = E_{n,m}^2 + (c \cdot \hbar K)^2$$



$$dK = \frac{1}{c\hbar} \frac{E}{\sqrt{E^2 - E_{n,m}^2}} dE$$

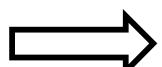


$$n_K = 2 \cdot dK / \Delta K$$

La densité d'états avec la même norme de K:

si  $E \geq E_{n,m}$

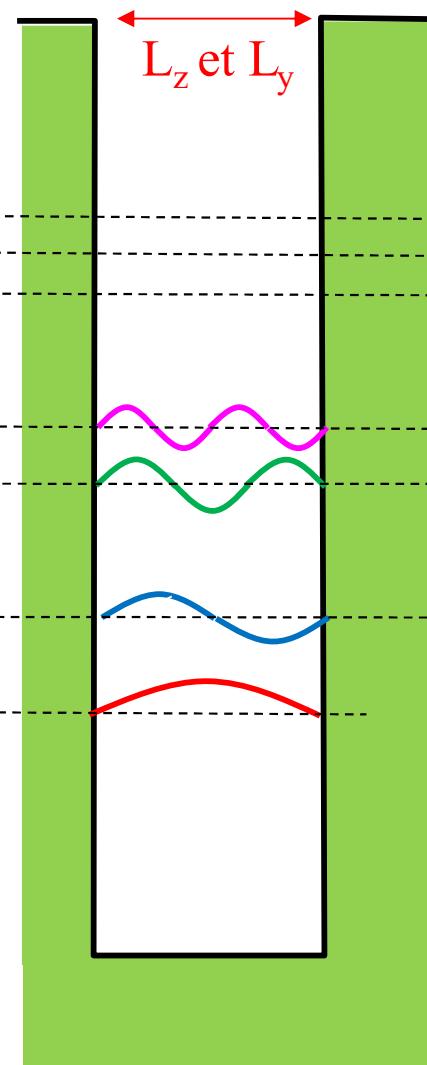
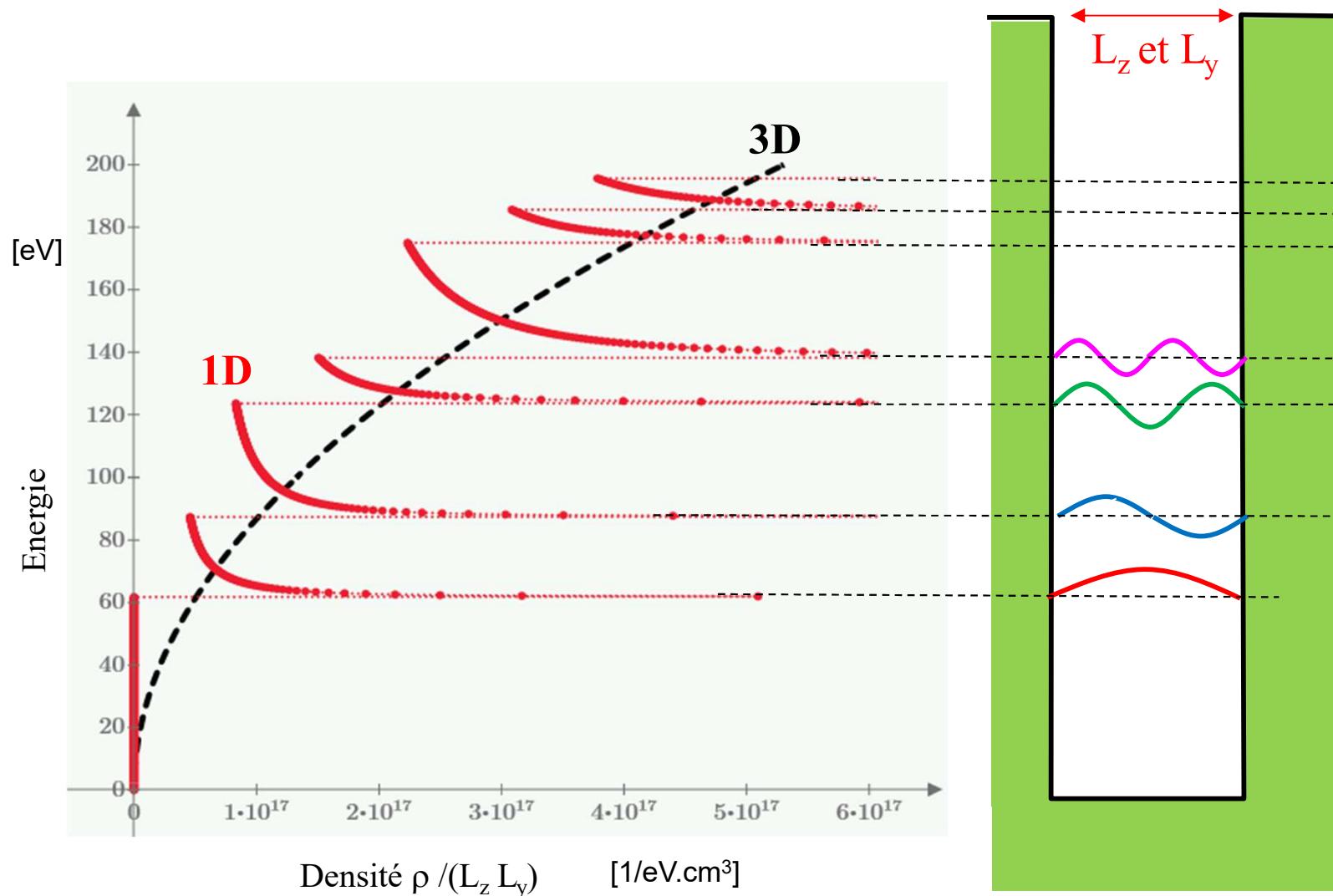
$$\rho_K \cdot dK \equiv \frac{n_K}{L} = \frac{1}{\pi} \cdot dK$$



$$\rho_{E,\gamma}^{1D} \cdot dE = \frac{1}{\pi} \cdot \frac{1}{c\hbar} \cdot \frac{E}{\sqrt{E^2 - E_{n,m}^2}} \cdot dE$$

$$\left[ \frac{1}{m} \right]$$

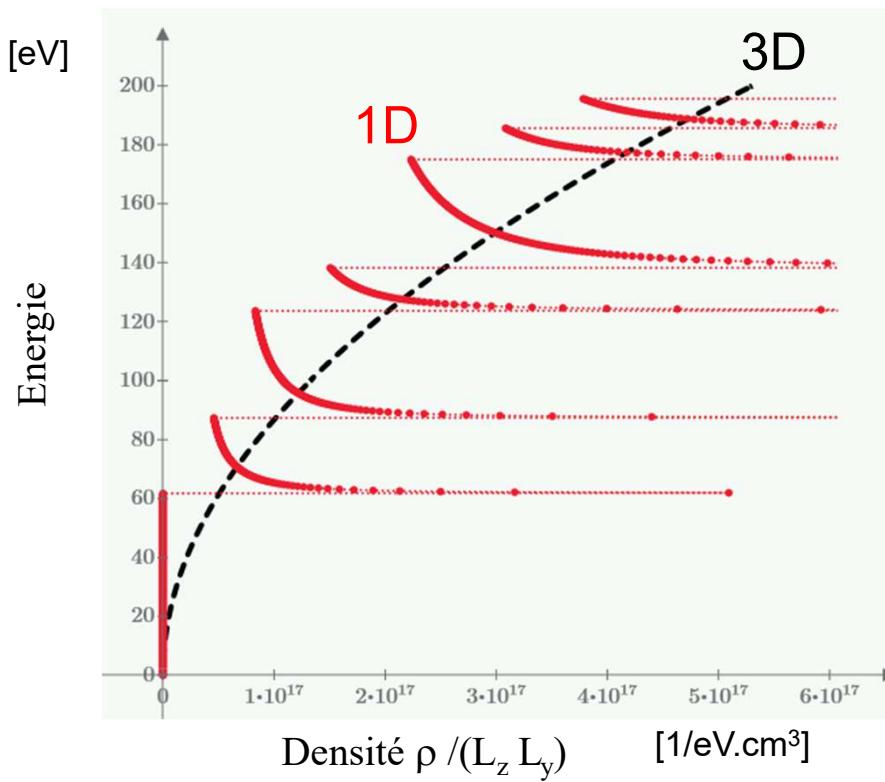
# Densité photonique 1D



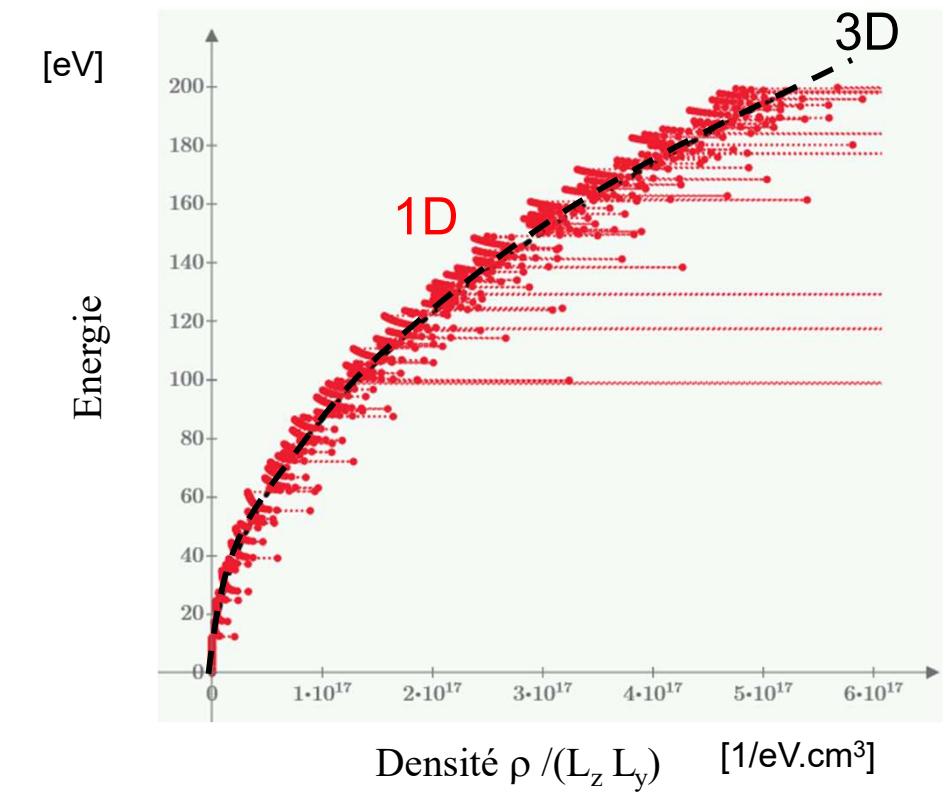
$$E_{n,m} = \hbar c \cdot \pi \cdot \sqrt{\left(\frac{n}{L_z}\right)^2 + \left(\frac{m}{L_y}\right)^2}$$

# Densité d'états photoniques 1D

$$\frac{\rho_{tot}^{1D}(E)}{L_y L_z} = \frac{1}{\pi} \cdot \frac{1}{\hbar c} \cdot \frac{1}{L_y L_z} \cdot \left[ \sum_{m=1}^{\infty} 1 \frac{E}{\sqrt{E^2 - E_{0,m}^2}} H(E - E_{0,m}) + \sum_{n=1}^{\infty} 1 \frac{E}{\sqrt{E^2 - E_{n,0}^2}} H(E - E_{n,0}) + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} 2 \frac{E}{\sqrt{E^2 - E_{n,m}^2}} H(E - E_{n,m}) \right]$$



$L_z = L_y = 10 \text{ nm}$



$L_z = L_y = 50 \text{ nm}$

Electrons

3D

$$\rho_{E,e}^{3D} \approx \sqrt{E - E_0}$$

$$\rho_{E,e,tot}^{2D} \approx \sum_n 1 \cdot H(E - E_n)$$

2D

$$\rho_{E,e,tot}^{1D} \approx \sum_n \sum_m \frac{1}{\sqrt{E - E_{n,m}}} \cdot H(E - E_{n,m})$$

1D

Photons

$$\rho_{E,\gamma}^{3D} \approx E^2$$

$$\rho_{E,\gamma,tot}^{2D} \approx \sum_n E \cdot H(E - E_n)$$

$$\left[ \frac{1}{eV \cdot m^3} \right]$$

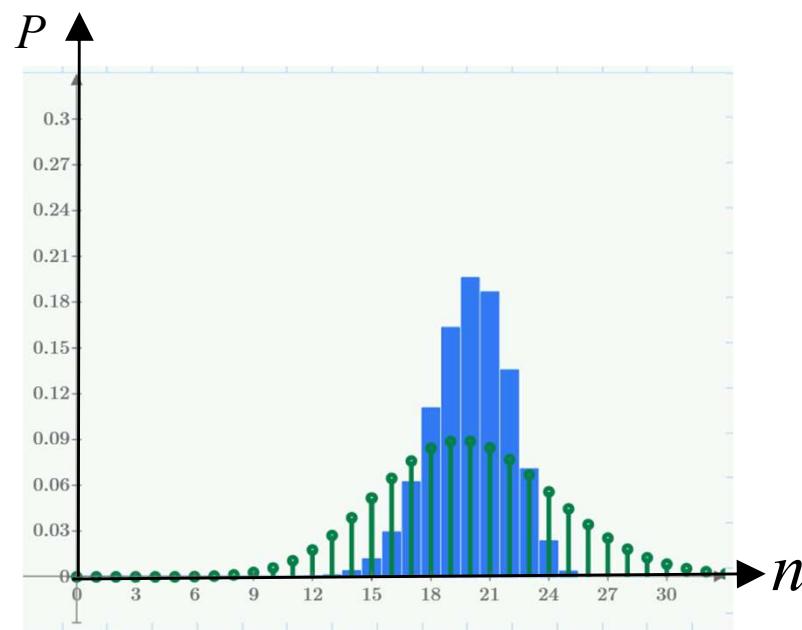
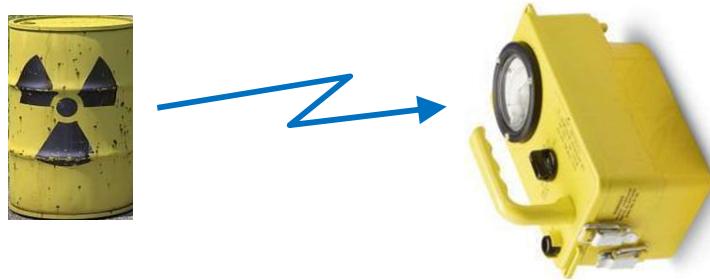
$$\left[ \frac{1}{eV \cdot m^2} \right]$$

$$\rho_{E,\gamma,tot}^{1D} \approx \sum_n \sum_m \frac{E}{\sqrt{E^2 - E_{n,m}^2}} H(E - E_{n,m})$$

$$\left[ \frac{1}{eV \cdot m} \right]$$

# **Statistiques et taux d'occupation**

# Exemples: événements identiques et indépendants



1 mesure par 0.1sec  
 $p=80\%$  de «bip»  
 $k= 25$  mesures

1 mesure par ms  
 $p=0.8\%$  de «bip»  
 $k= 2'500$  mesures

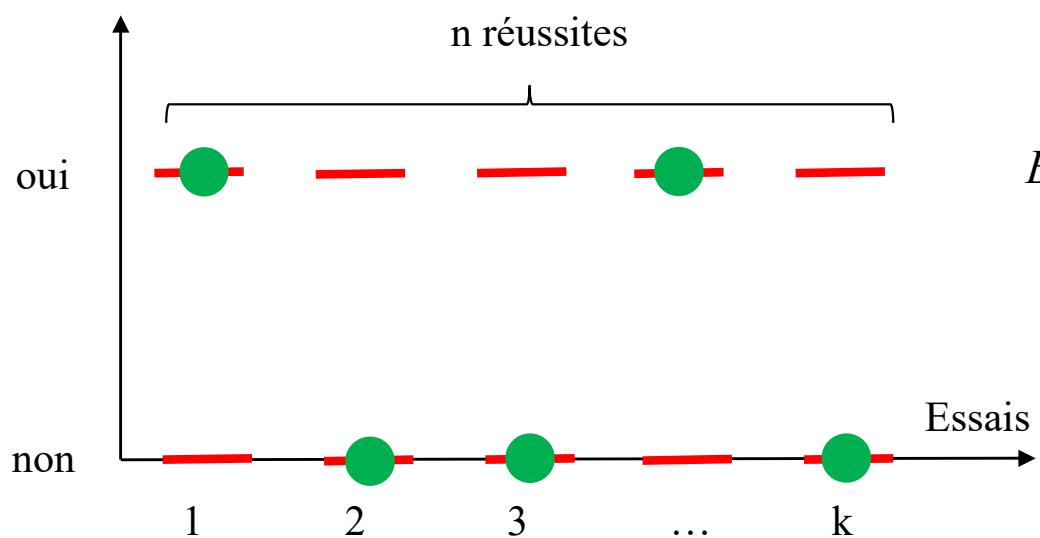
**Loi binomiale**

$p \searrow 0$   
 $k \nearrow \infty$   
 $\langle n \rangle = p \cdot k$  constant

**Statistique de Poisson**

# Loi binomiale

- k essais indépendants
- Chaque essai a une probabilité p de réussite



Probabilité d'obtenir n réussites = loi binomiale

$$B(n) = \binom{k}{n} p^n \cdot (1-p)^{k-n} = \frac{k \cdot (k-1) \cdots (k-n+1)}{n!} p^n \cdot (1-p)^{k-n}$$

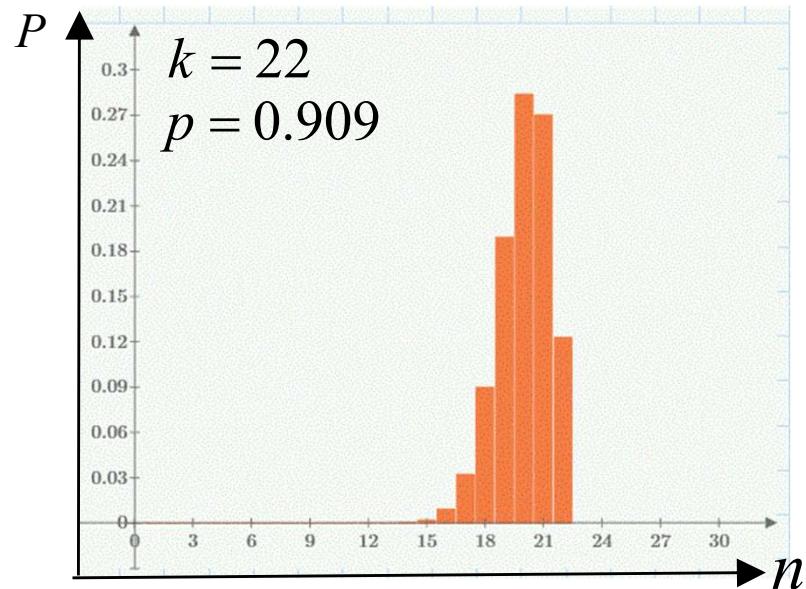
↑  
combinaison

Moyenne des réussites:  $\langle n \rangle = k \cdot p$

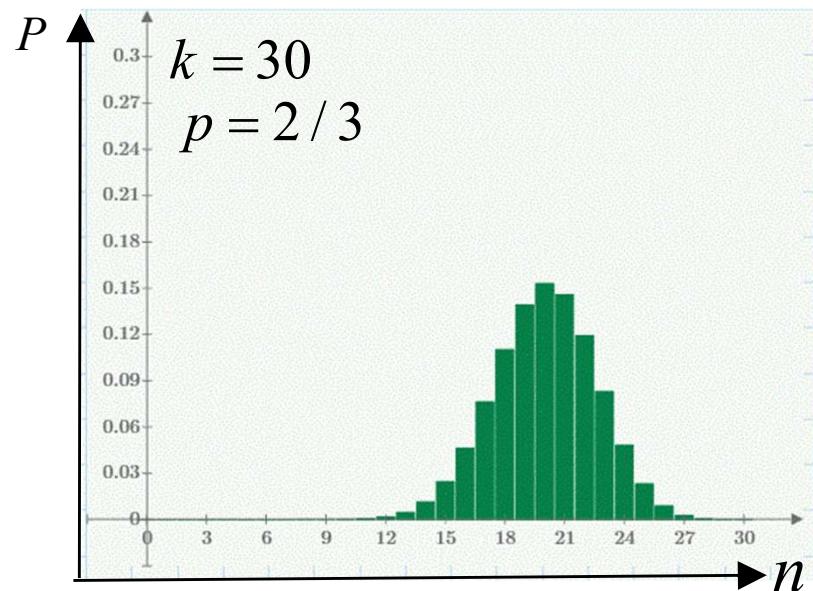
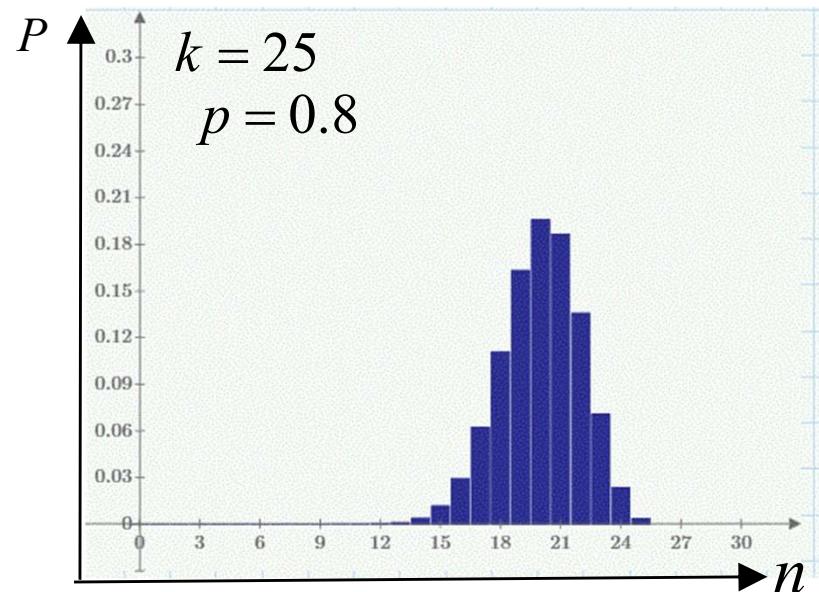
Variance des réussites:

$$\Delta n^2 = \langle n \rangle \cdot (1 - p)$$

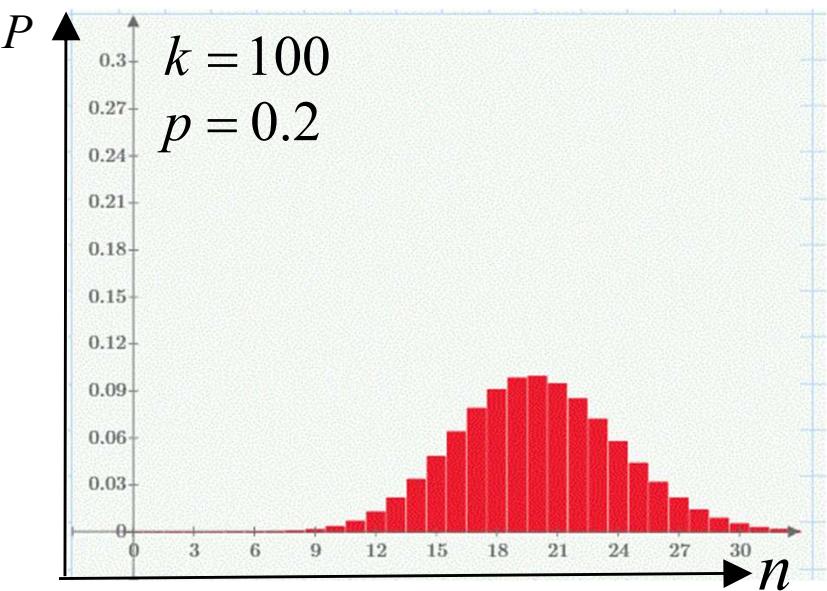
## Exemples: binomiale $\langle n \rangle = 20$



$$\langle n \rangle = 20$$



$$p \equiv \frac{\langle n \rangle}{k}$$



# Poisson distribution

We use the limits:  
(Poisson distribution)

$$P(n) = \lim_{\substack{k \rightarrow \infty \\ p \rightarrow \langle n \rangle / k}} B(n)$$

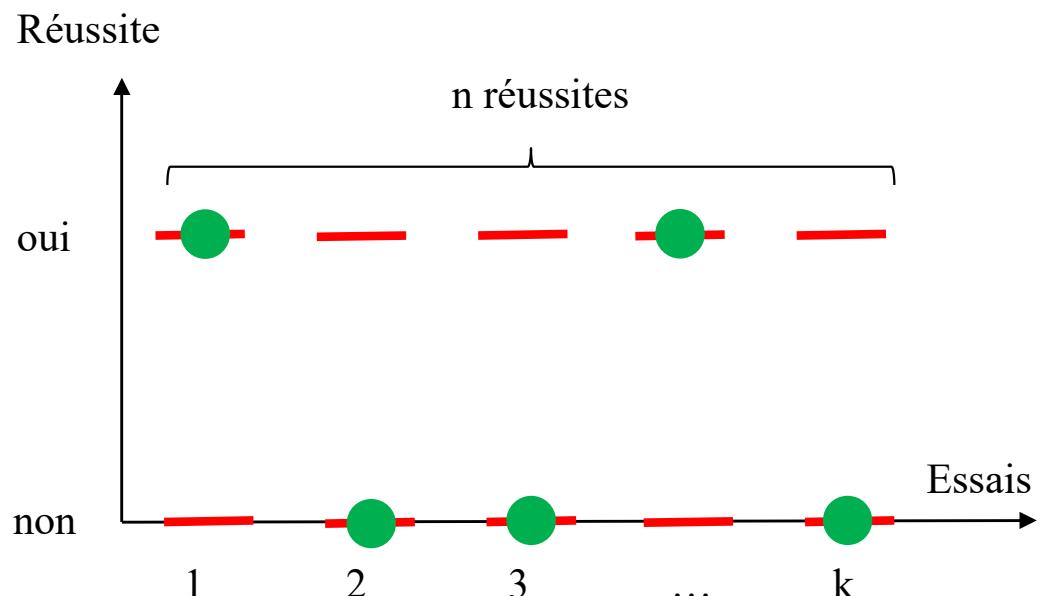
- The number  $k$  of trials tends to infinity
- The probability  $p$  tends to zero
- The mean number detected is  $k.p$

$$P(n) = \lim_{\substack{k \rightarrow \infty \\ p \rightarrow \langle n \rangle / k}} \frac{k.(k-1)...(k-n+1)}{n!} p^n \cdot (1-p)^{k-n} = \lim_{\substack{k \rightarrow \infty \\ p \rightarrow \langle n \rangle / k}} \frac{1}{n!} \cdot \frac{k}{k} \frac{(k-1)}{k} \dots \frac{(k-n+1)}{k} (kp)^n \cdot (1-p)^k \cdot \frac{1}{(1-p)^n}$$

$$P(n) = \lim_{\substack{k \rightarrow \infty \\ p \rightarrow \langle n \rangle / k}} \frac{1}{n!} \cdot \langle n \rangle^n \cdot \left[ (1-p)^{1/p} \right]^{\langle n \rangle} = \frac{1}{n!} \cdot \langle n \rangle^n \cdot e^{-\langle n \rangle}$$

Poisson distribution

$$P(n) = \frac{1}{n!} \cdot \langle n \rangle^n \cdot e^{-\langle n \rangle}$$



Moyenne du nombre de réussites:

$$\langle n \rangle = \sum_{n=0}^{\infty} n \cdot P(n)$$

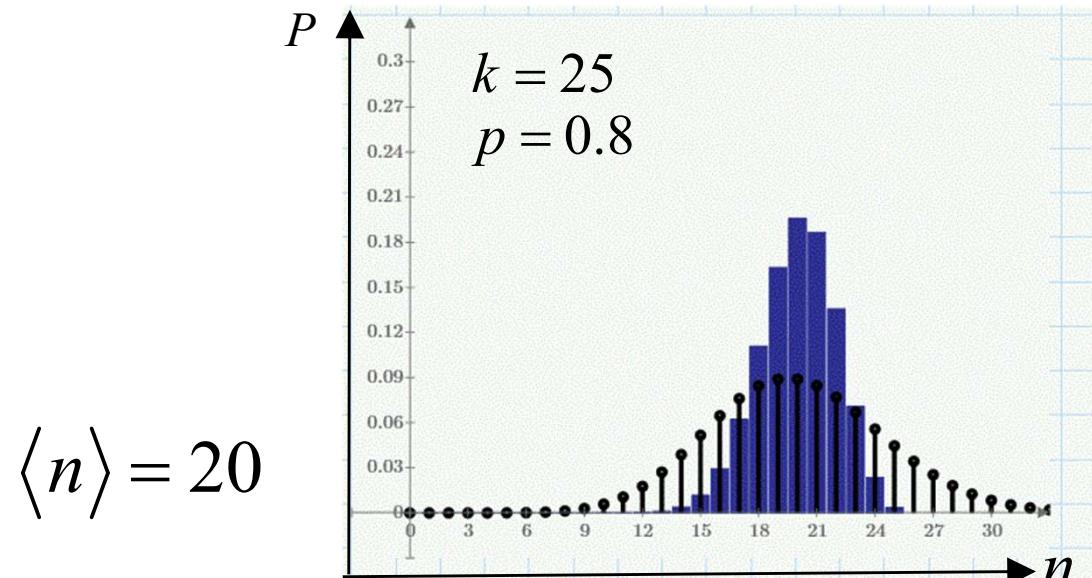
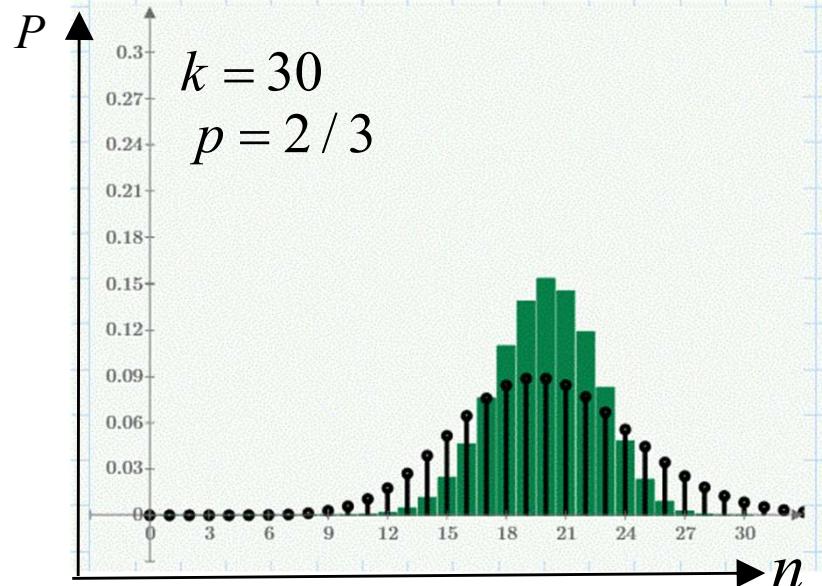
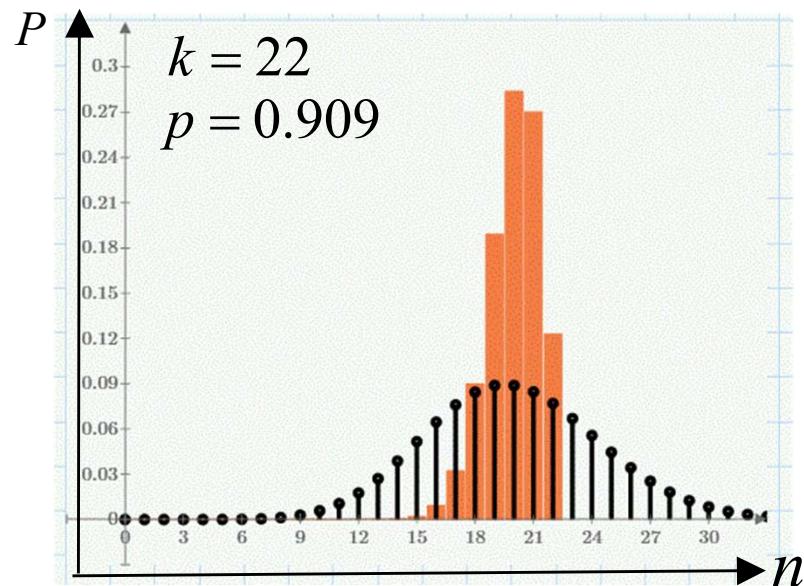
Moment quadratique du nombre de réussites:

$$\langle n^2 \rangle = \sum_{n=0}^{\infty} n^2 \cdot P(n) = \langle n \rangle^2 + \langle n \rangle$$

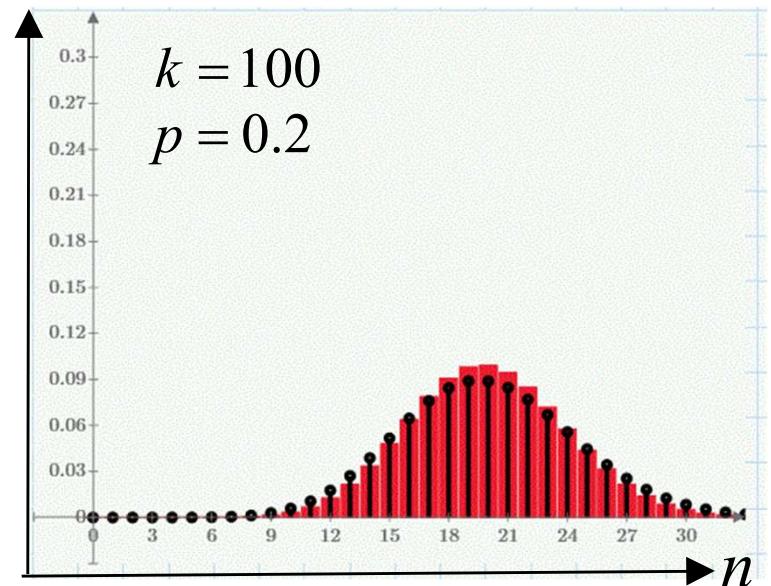
Variance du nombre de réussites:

$$\Delta n^2 \equiv \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle$$

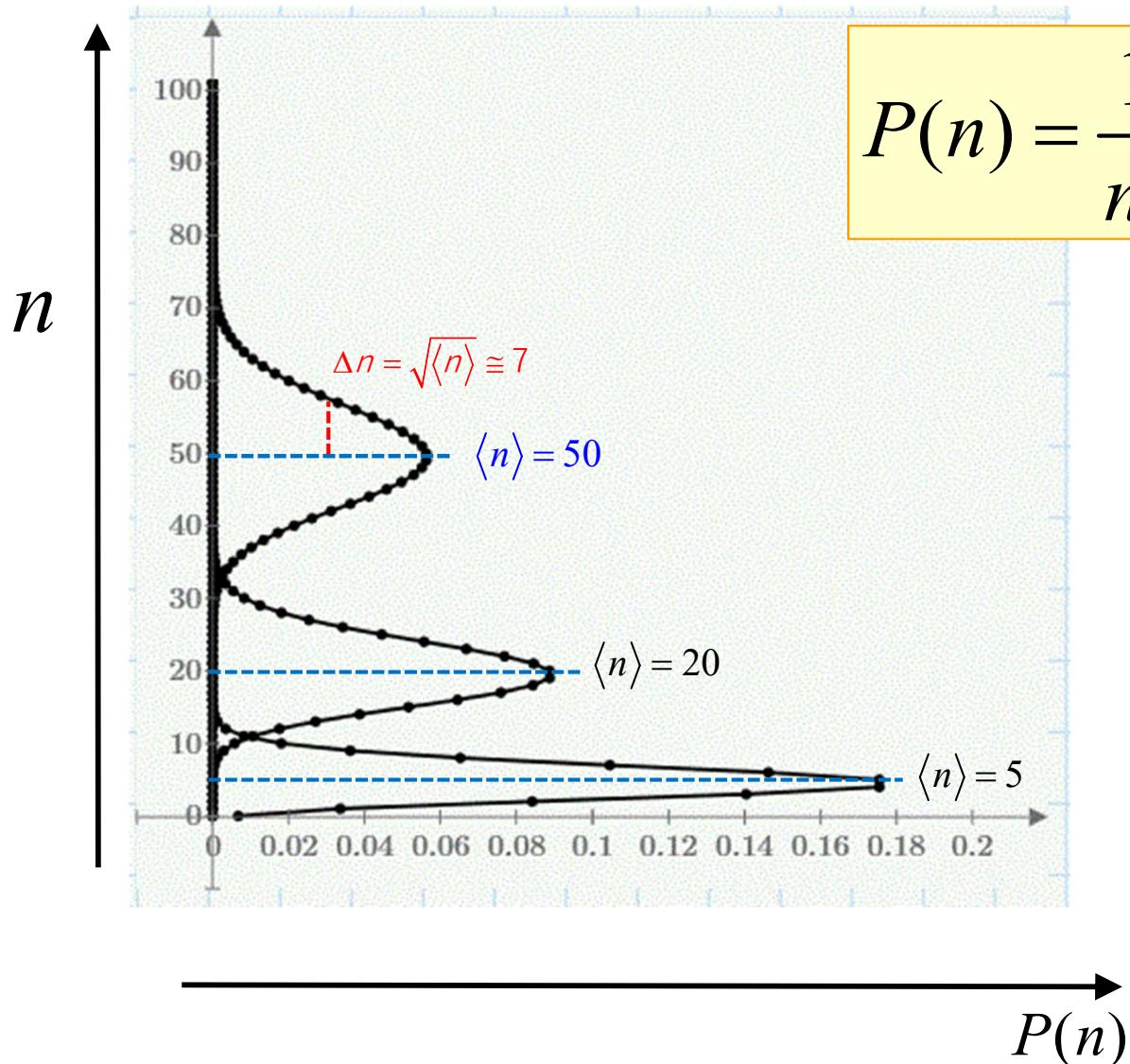
# Exemples: Binomiale et Poisson $\langle n \rangle = 20$



$$p \equiv \frac{\langle n \rangle}{k}$$



## Exemples: Poisson

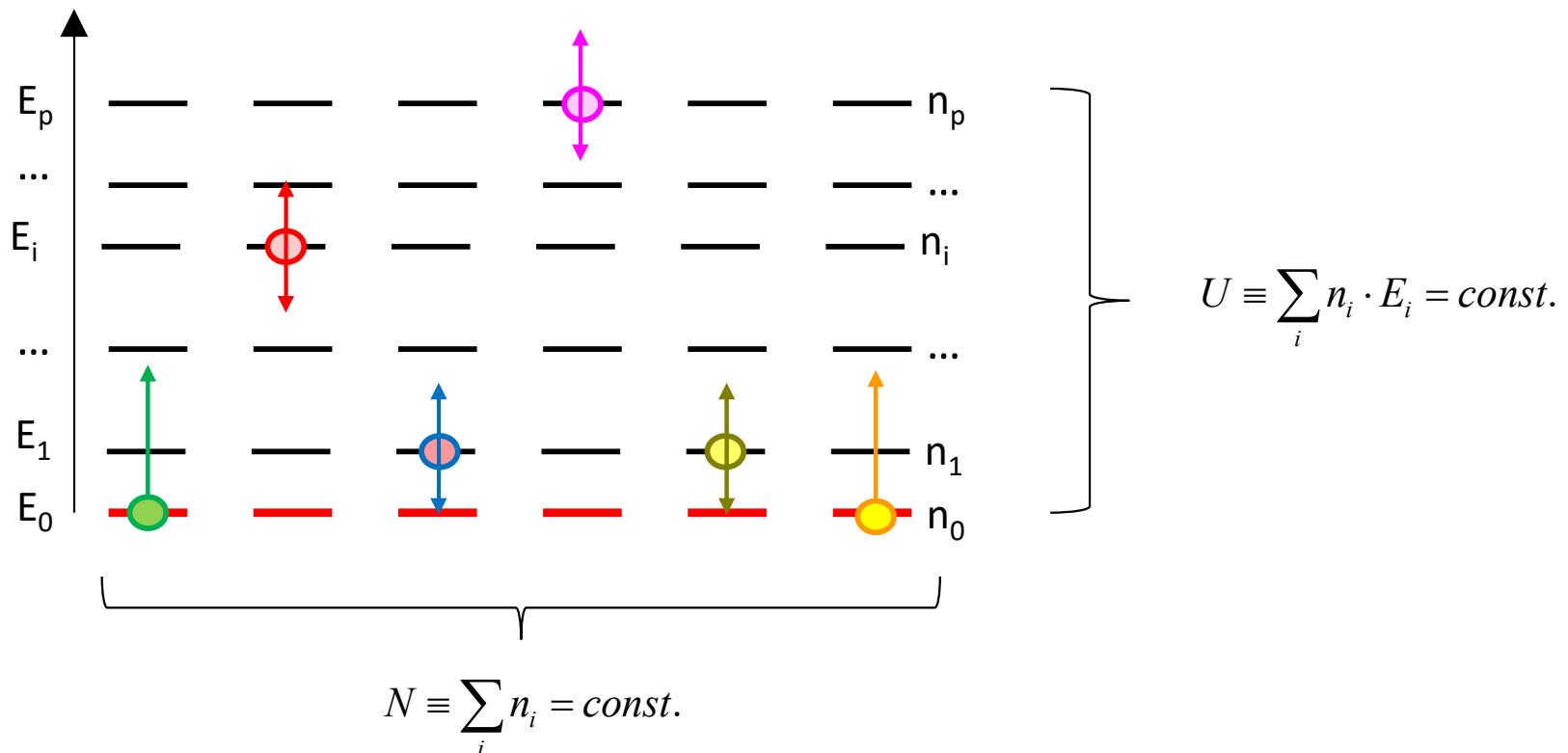


$$P(n) = \frac{1}{n!} \cdot \langle n \rangle^n \cdot e^{-\langle n \rangle}$$

$$\Delta n = \sqrt{\langle n \rangle}$$

# **Statistique de Boltzmann**

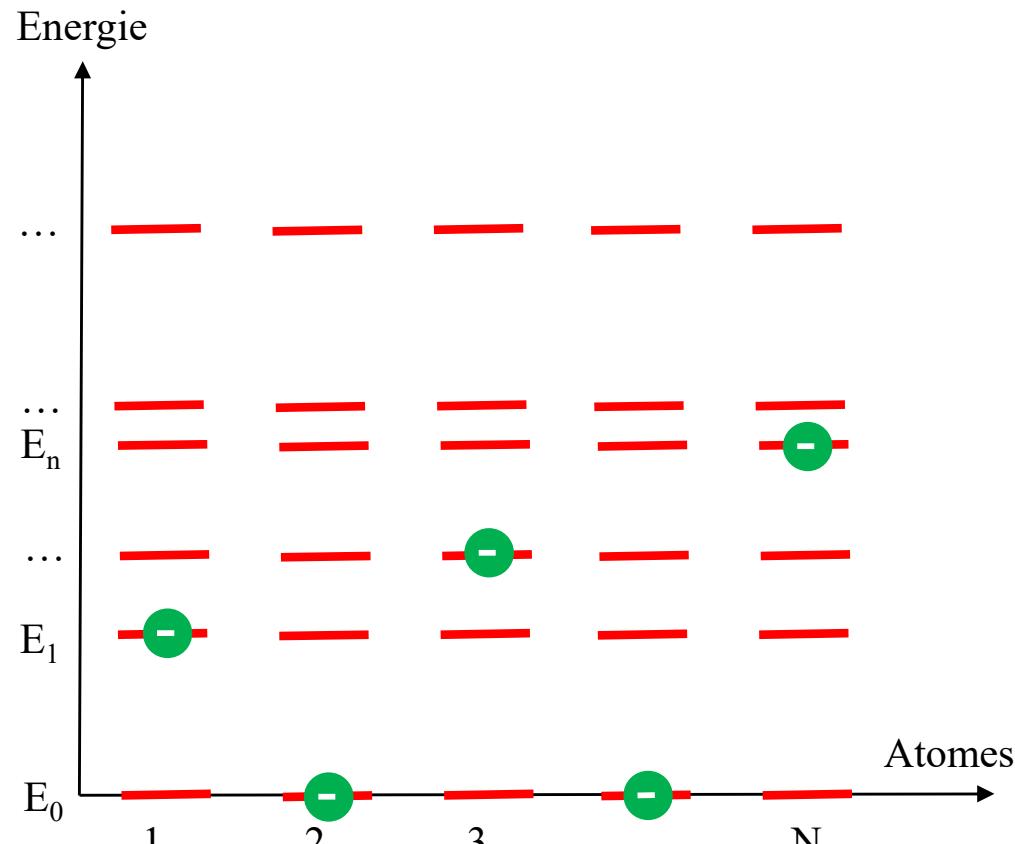
# Statistique de Boltzmann



Toutes les particules sont distinctes mais identiques.  
Les niveaux d'énergie sont identiques.

Toutes les configurations ont la même probabilité, mais elles doivent maintenir constants le nombre d'atomes et l'énergie totale.

# Statistique de Boltzmann pour des atomes



Probabilité d'occupation de l'état d'énergie  $E_n$ :

$$f_b(E_n) = \alpha \cdot e^{-\frac{E_n}{kT}} = \frac{e^{-\frac{E_n}{kT}}}{\sum_{j=0}^{\infty} e^{-\frac{E_j}{kT}}}$$

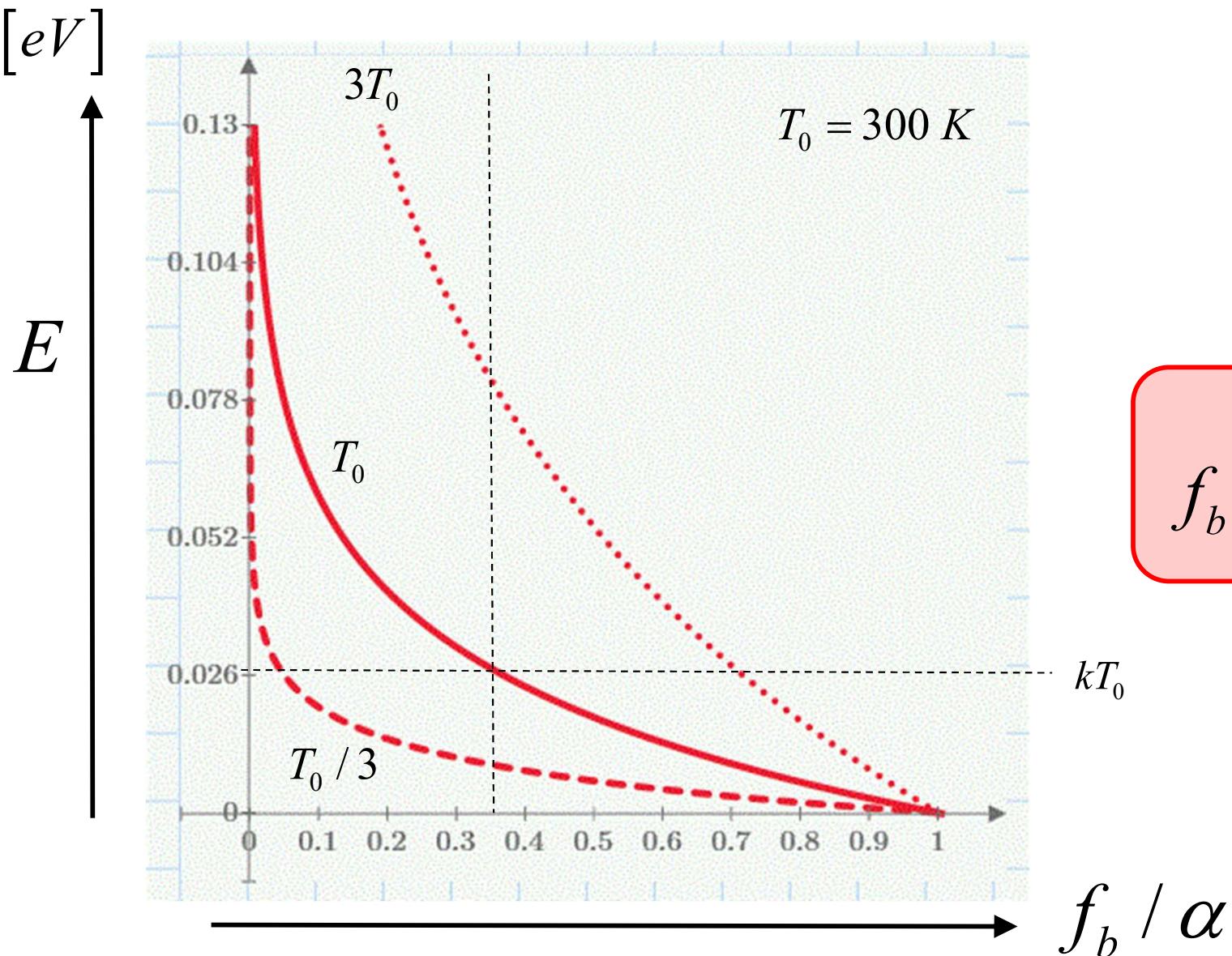
Moyenne de l'énergie d'un atome:

$$\langle E \rangle = \sum_{j=0}^{\infty} E_j \cdot f_b(E_j)$$

Second moment de l'énergie d'un atome:

$$\langle E^2 \rangle = \sum_{j=0}^{\infty} E_j^2 \cdot f_b(E_j)$$

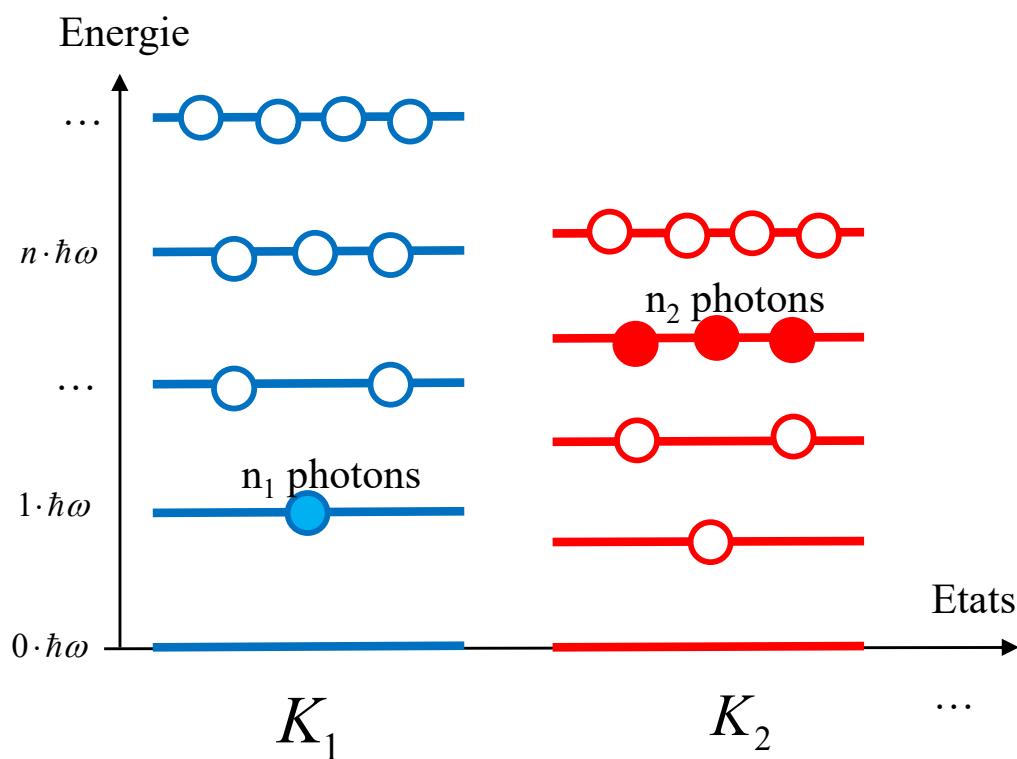
## Exemples: Boltzmann



$$f_b(E) = \alpha \cdot e^{-\frac{E}{kT}}$$

# **Statistique de Bose-Einstein**

## Répartition régulière des niveaux



Probabilité d'occupation d'un état d'énergie:

$$f_b(n \cdot \hbar\omega) = \alpha \cdot e^{-\frac{n \cdot \hbar\omega}{kT}} = \frac{e^{-\frac{n \cdot \hbar\omega}{kT}}}{\sum_{j=0}^{\infty} e^{-\frac{j \cdot \hbar\omega}{kT}}}$$

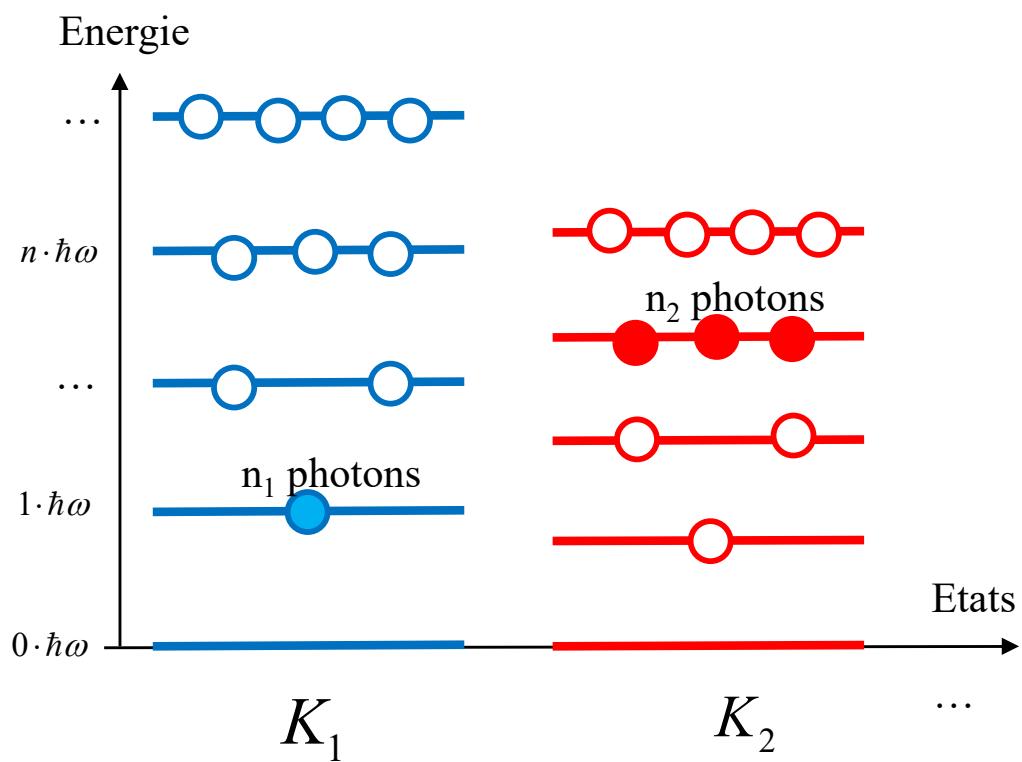
**Statistique de Boltzmann (b)**

Moyenne du nombre de photons dans un mode K:

$$F_\gamma(E) \equiv \langle n \rangle_{BE} = \frac{\sum_{n=0}^{\infty} n \cdot e^{-\frac{n \cdot \hbar\omega}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{n \cdot \hbar\omega}{kT}}} = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$$

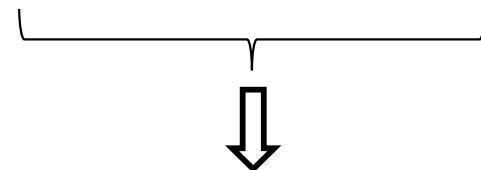
**Statistique de Bose-Einstein (BE)**

## Répartition régulière des niveaux



$$f_b(n \cdot \hbar\omega) = \frac{e^{-\frac{n \cdot \hbar\omega}{kT}}}{\sum_{j=0}^{\infty} e^{-\frac{j \cdot \hbar\omega}{kT}}} = e^{-\frac{n \cdot \hbar\omega}{kT}} \cdot \left(1 - e^{-\frac{\hbar\omega}{kT}}\right)$$

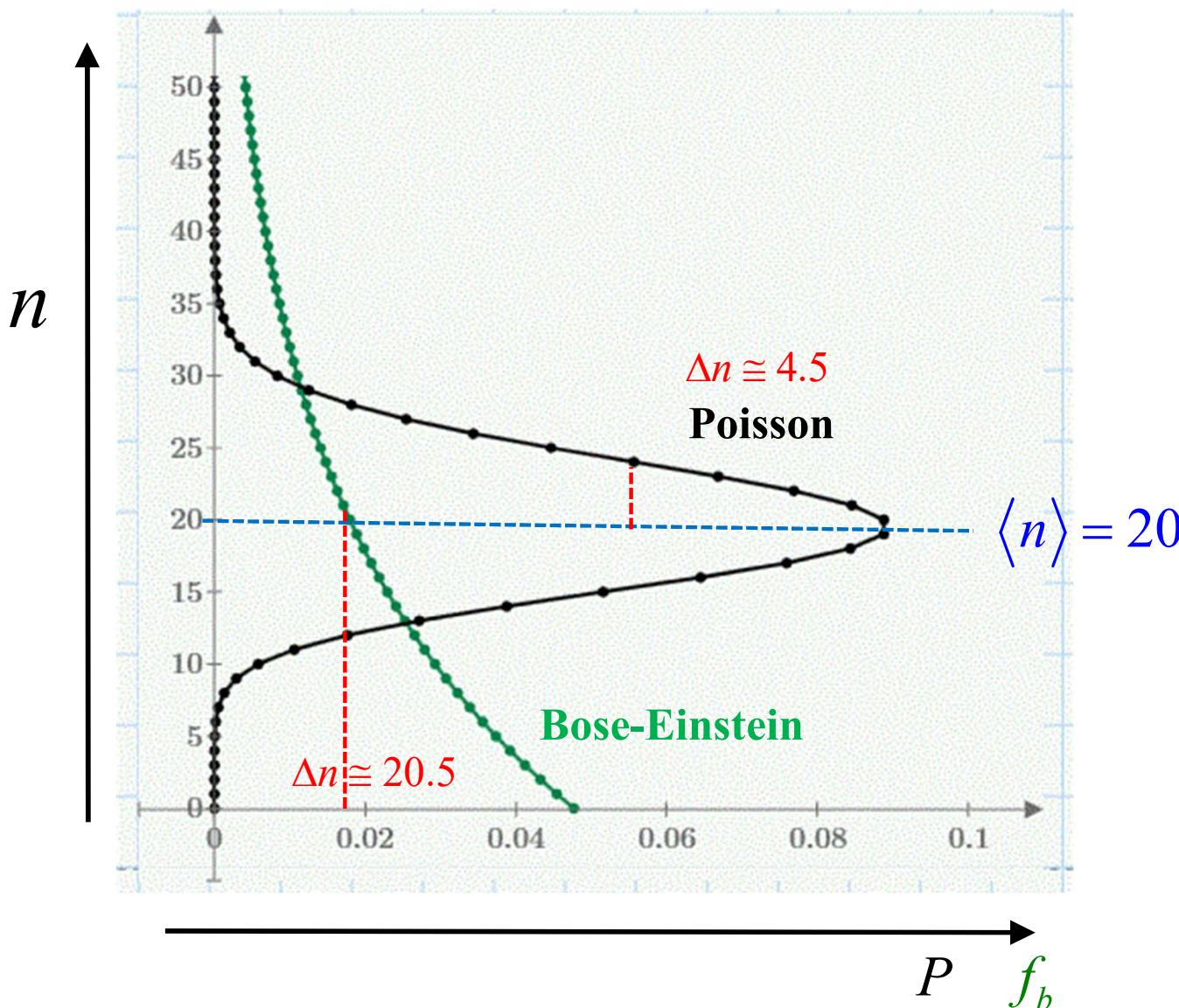
$$\langle n \rangle_{BE} = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$$



Probabilité d'occupation d'un état d'énergie:

$$f_b(n) = \frac{\langle n \rangle^n}{(\langle n \rangle + 1)^{n+1}}$$

# Bose-Einstein comparé à Poisson



$$P(n) = \frac{1}{n!} \cdot \langle n \rangle^n \cdot e^{-\langle n \rangle}$$

$$\Delta n = \sqrt{\langle n \rangle}$$

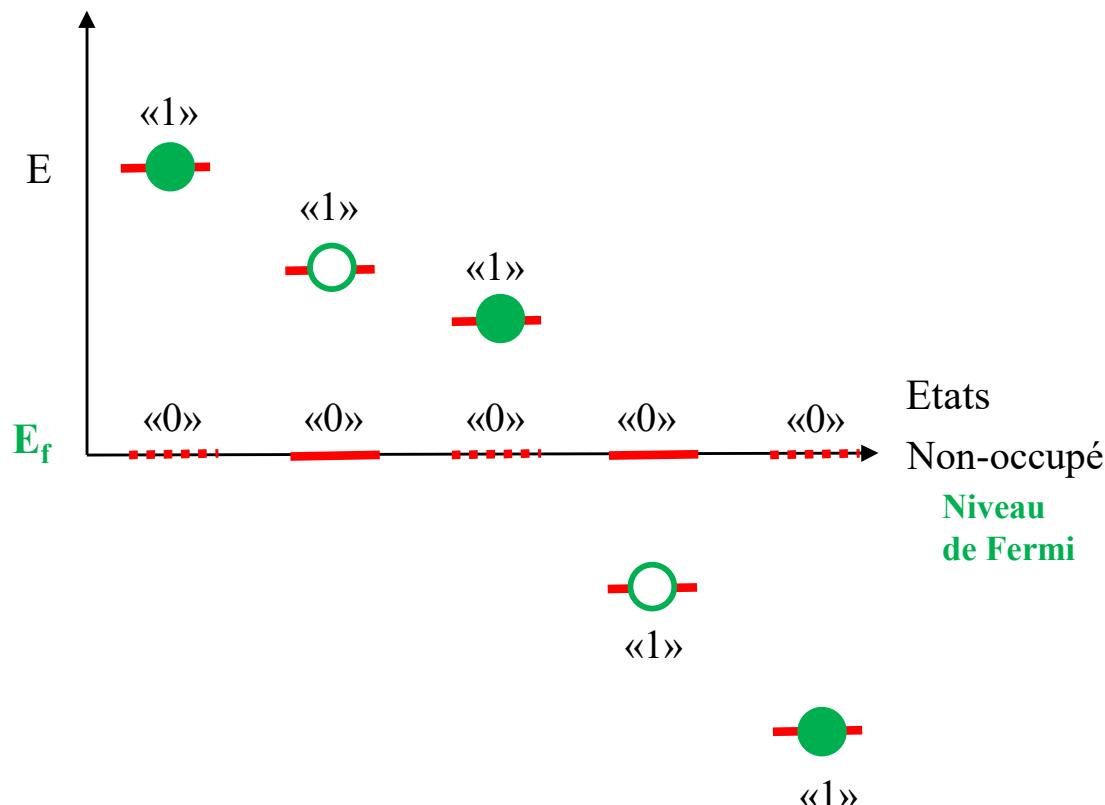
$$f_b(n) = \frac{\langle n \rangle^n}{(\langle n \rangle + 1)^{n+1}}$$

$$\Delta n = \sqrt{\langle n \rangle \cdot (\langle n \rangle + 1)}$$

# Statistique de Fermi-Dirac

# Statistique de Fermi-Dirac

- État non-occupé ou occupé par un seul électron
- Chaque état a une probabilité  $p$  d'être occupé



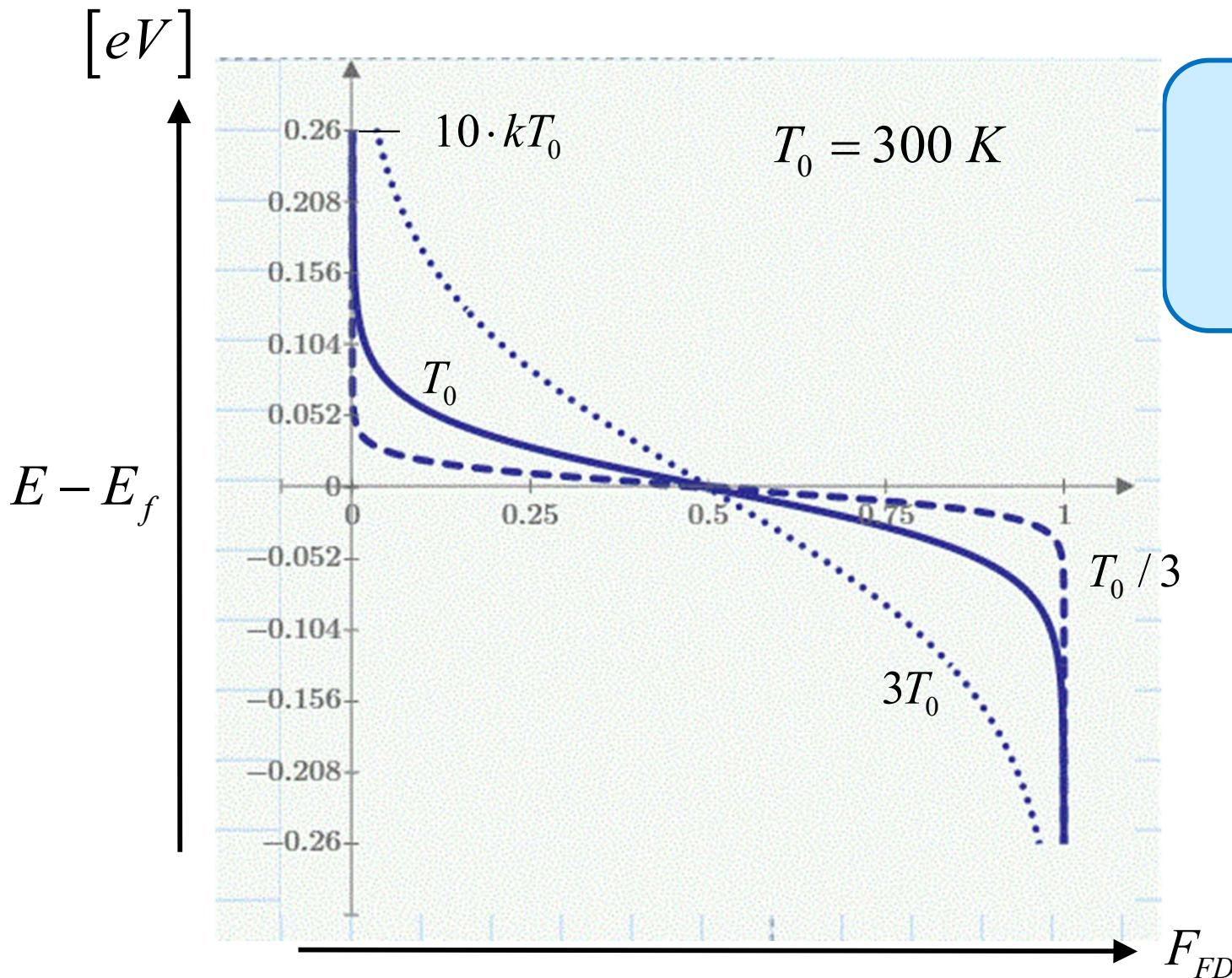
Probabilité d'occupation:

$$p = f_b(E) = \alpha \cdot e^{-\frac{E}{kT}} = \frac{e^{-\frac{E}{kT}}}{e^{-\frac{E_f}{kT}} + e^{-\frac{E}{kT}}}$$

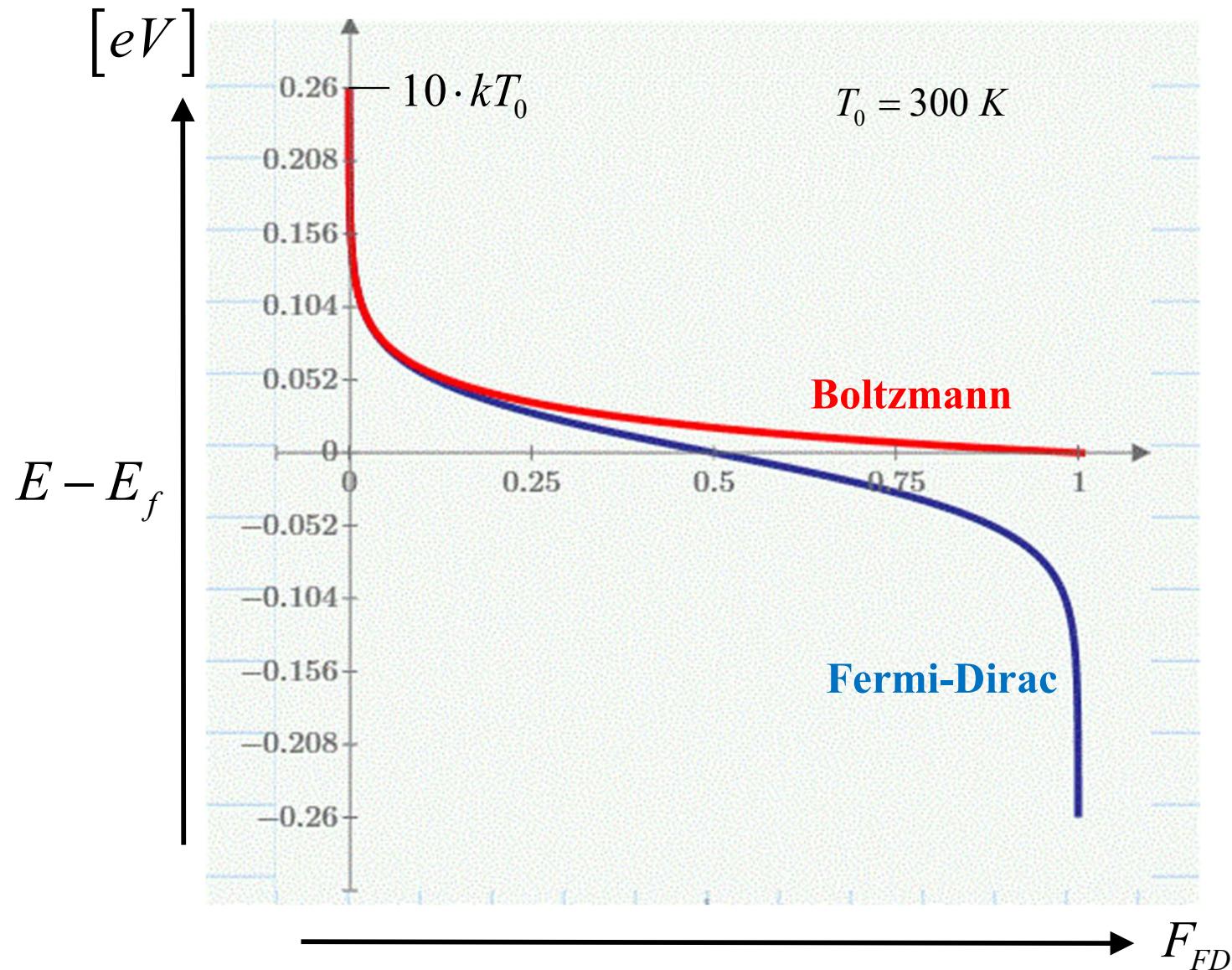
$$F_e(E) \equiv F_{FD} = \frac{1}{1 + e^{\frac{(E-E_f)}{kT}}}$$

**Statistique de Fermi-Dirac (FD)**

## Exemples: Fermi-Dirac



$$F_{FD} = \frac{1}{1 + e^{\frac{(E - E_f)}{kT}}}$$



$$f_b(E) = e^{-\frac{E-E_f}{kT}}$$

$$F_{FD} = \frac{1}{1 + e^{\frac{(E-E_f)}{kT}}}$$

# **La science quantique**

## **Une vision singulière**

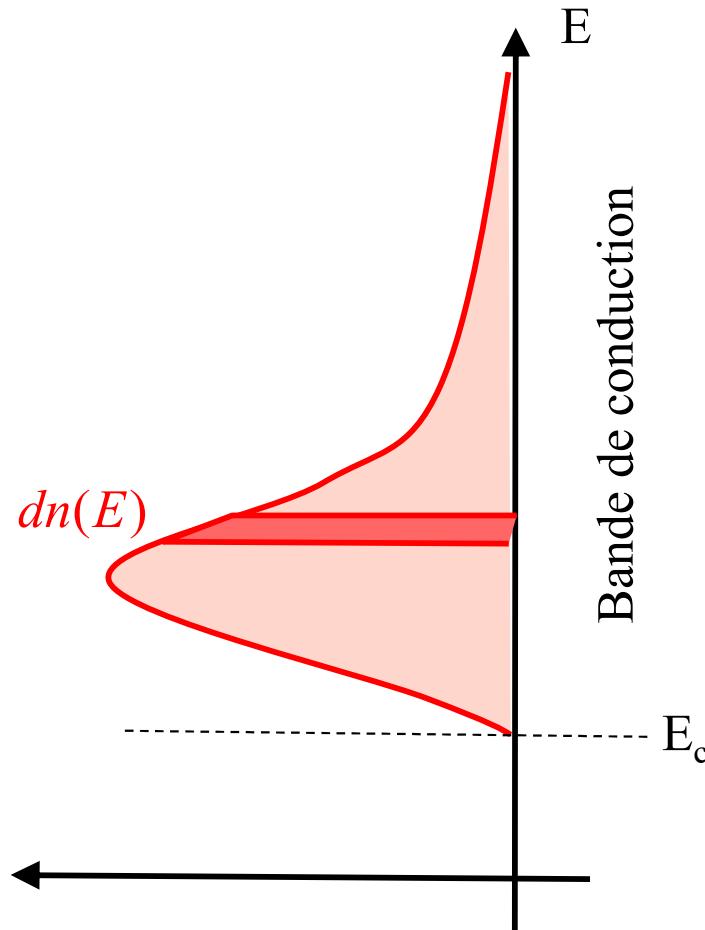
### **Applications**

P.A. Besse

# Semiconducteurs: Concentrations de porteurs

# Principe de calcul

Electrons



Densité d'électrons libres dans un semi-conducteur:

$$n = \int_{E_c}^{\infty} dn(E)$$

$$n = \int_{E_c}^{\infty} (2) \cdot \rho_e^{3D}(E) \cdot F_e(E) \cdot dE$$

Arrows pointing to the terms in the equation:
 

- $E_c$ : Contribution d'un état
- $\rho_e^{3D}(E)$ : Densité d'états
- $F_e(E)$ : Taux d'occupation

# Concentration d'électron libre et de trous

Concentration d'électrons libre dans les bandes de conduction:

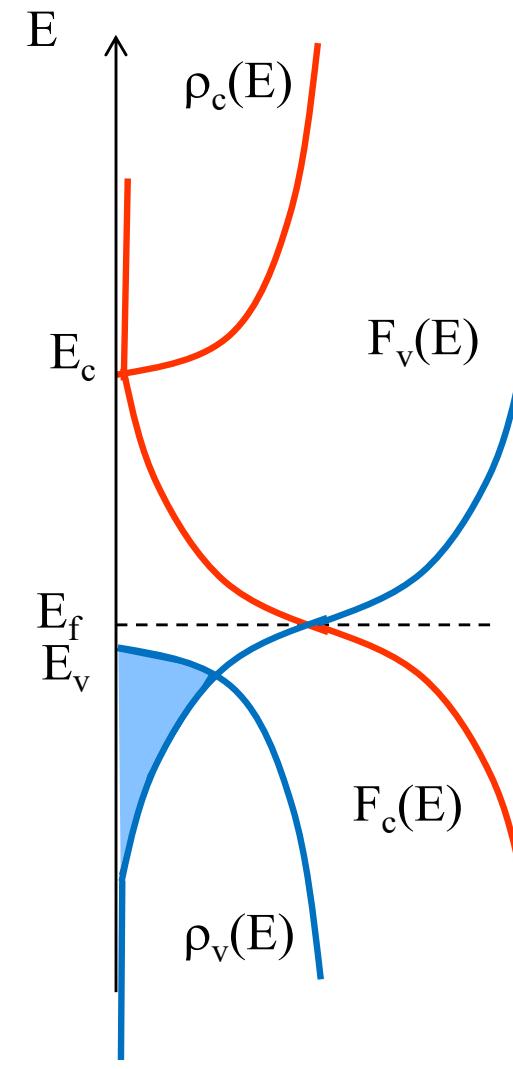
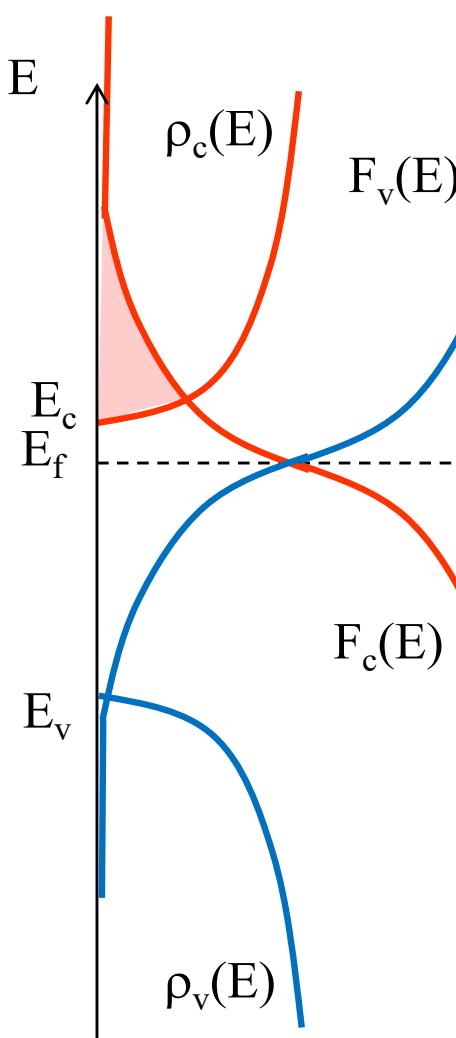
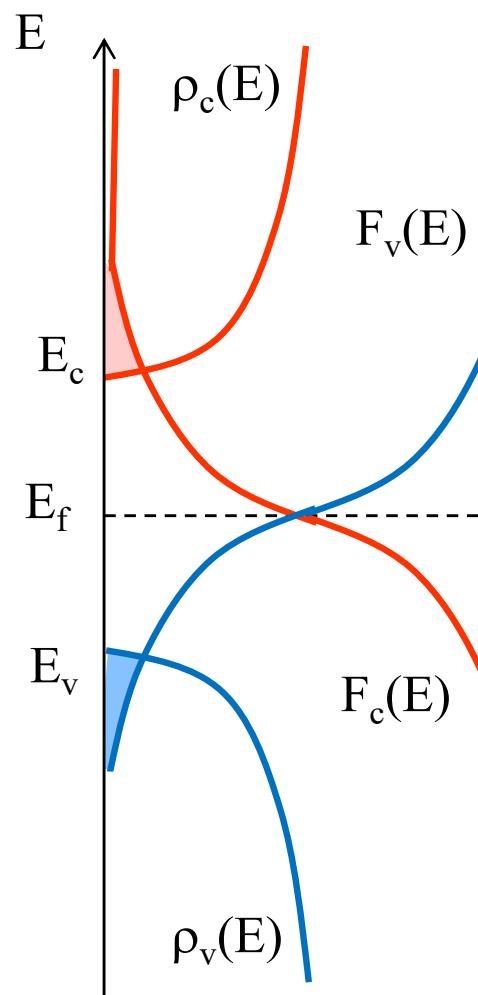
$$n = 2 \cdot \int_{E_c}^{E_{c,\max}} \rho_c(E) \cdot F_c(E) \cdot dE = \sqrt{2} \frac{\left(m_{dos,n}^*\right)^{3/2}}{\pi^2 \hbar^3} \int_{E_c}^{\infty} \sqrt{E - E_c} \cdot \frac{1}{1 + e^{(E - E_F)/kT}} \cdot dE$$

Concentration de trous dans les bandes de valence:

$$p = 2 \cdot \int_{E_{v,\min}}^{E_v} \rho_v(E) \cdot F_v(E) \cdot dE = \sqrt{2} \frac{\left(m_{dos,p}^*\right)^{3/2}}{\pi^2 \hbar^3} \int_{-\infty}^{E_v} \sqrt{E_v - E} \cdot \frac{1}{1 + e^{(E_F - E)/kT}} \cdot dE$$

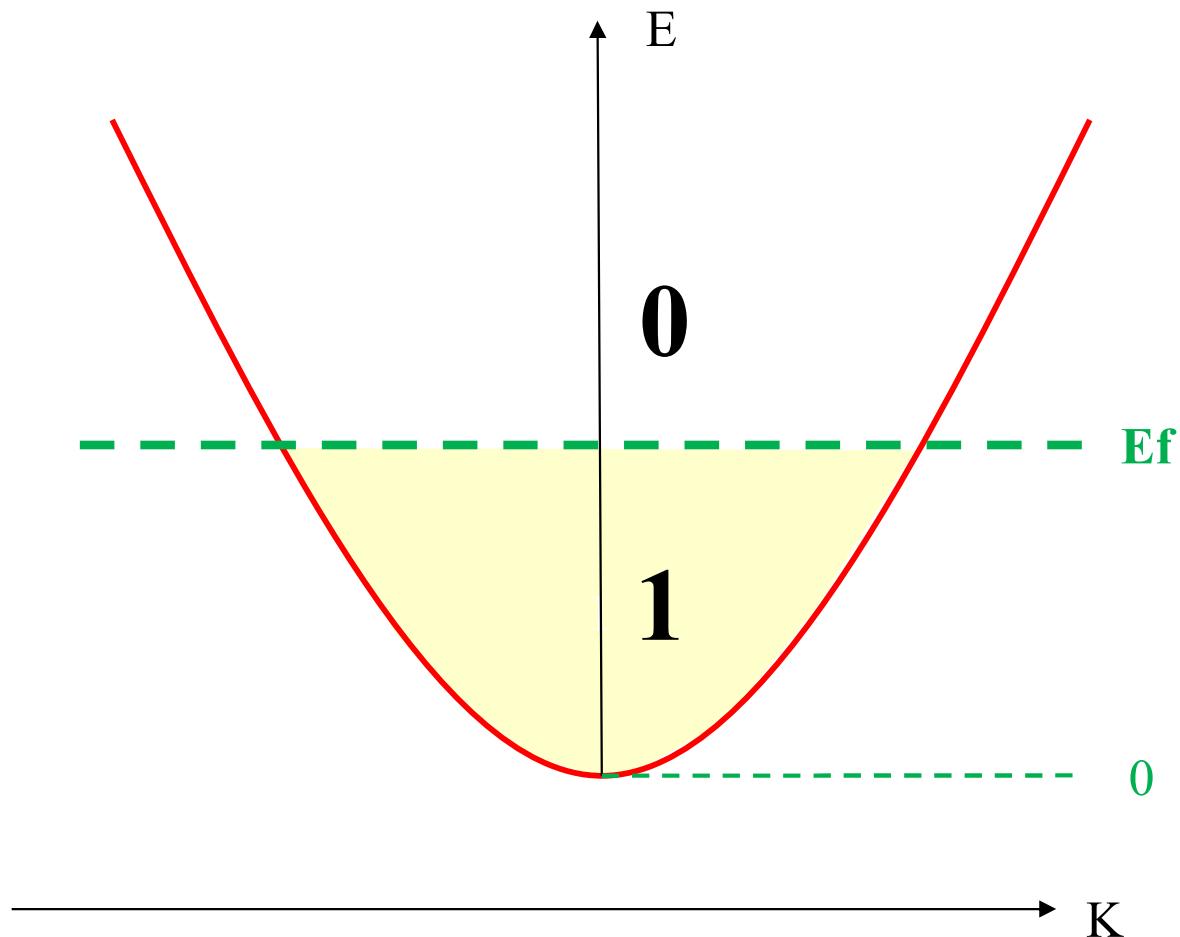
- Remarques:
- Ces équations sont aussi valides pour un semi-conducteur dopé
  - Un paramètre est pour l'instant libre: l'énergie de Fermi  $E_F$ .

# Concentration: résolution graphique



# Métaux: Concentration de porteurs à T=0K

# Schéma de bande paraboloïque



Densité d'état:

$$\rho(E) \approx \sqrt{E} \cdot dE$$

$T=0K$

Taux d'occupation =

Concentration de porteurs libres à T=0K:

$$n = \frac{N}{V} = \int_0^{E_F} 2\rho(E) \cdot 1 \cdot dE = \frac{\sqrt{2m^3}}{\pi^2 \hbar^3} \int_0^{E_F} \sqrt{E} dE = \frac{(2m)^{3/2}}{3\pi^2 \hbar^3} \cdot E_F^{3/2}$$

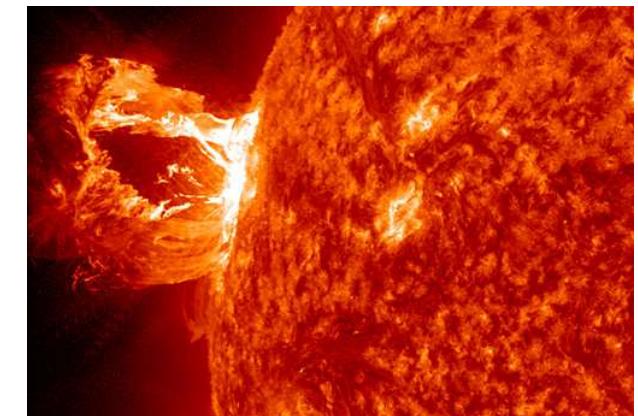
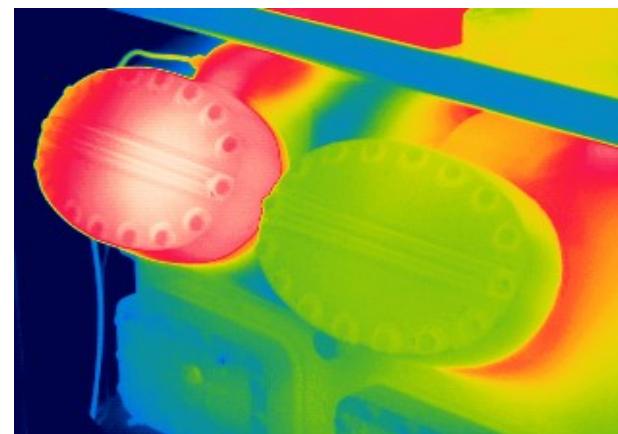
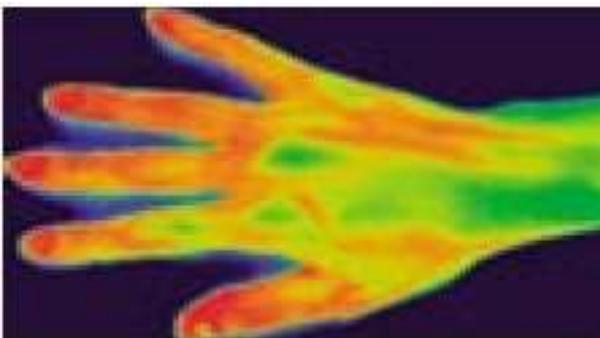
Energie cinétique moyenne d'un électron à T=0K:

$$n \cdot \langle E \rangle = \int_0^{E_F} 2 \cdot E \cdot \rho(E) \cdot 1 \cdot dE = \frac{\sqrt{2m^3}}{\pi^2 \hbar^3} \int_0^{E_F} E^{3/2} \cdot dE = \frac{(2m)^{3/2}}{5\pi^2 \hbar^3} \cdot E_F^{5/2} = \frac{3}{5} n \cdot E_F$$

**Photons et  
loi de Planck**

# Black Body

The spectrum of a black body depends only on its temperature and geometry



# Puissance spectrale par surface et angle d'émission

## Principe de calcul:

$$dP_E \equiv \frac{c}{4\pi} \cdot dW_E = \frac{c}{4\pi} \cdot (2 \cdot E) \cdot \rho_{\gamma}^{3D} \cdot F_{\gamma}(E) \cdot dE$$

Puissance spectrale par surface et angle d'émission

Energie spectrale par volume

Energie du photon

Taux d'occupation

Densité d'états

Polarisation

$$\rho_{\gamma}^{3D}(E) = \frac{1}{2\pi^2} \cdot \frac{E^2}{c^3 \hbar^3}$$

$$F_{\gamma}(E) = \frac{1}{e^{\frac{E}{kT}} - 1}$$

**Loi du corps noir de Planck**

$$dP_E = \frac{2}{c^2 h^3} \cdot E^3 \cdot \frac{1}{e^{\frac{E}{kT}} - 1} \cdot dE$$

**Puissance totale par surface S et angle d'émission  $\Omega$ :**

$$\frac{P_{tot}}{S \cdot \Omega} = \int_0^{\infty} dP_E = \frac{2}{c^2 h^3} \cdot \int_0^{\infty} \frac{E^3}{e^{\frac{E}{kT}} - 1} \cdot dE = \frac{\sigma}{\pi} \cdot T^4$$

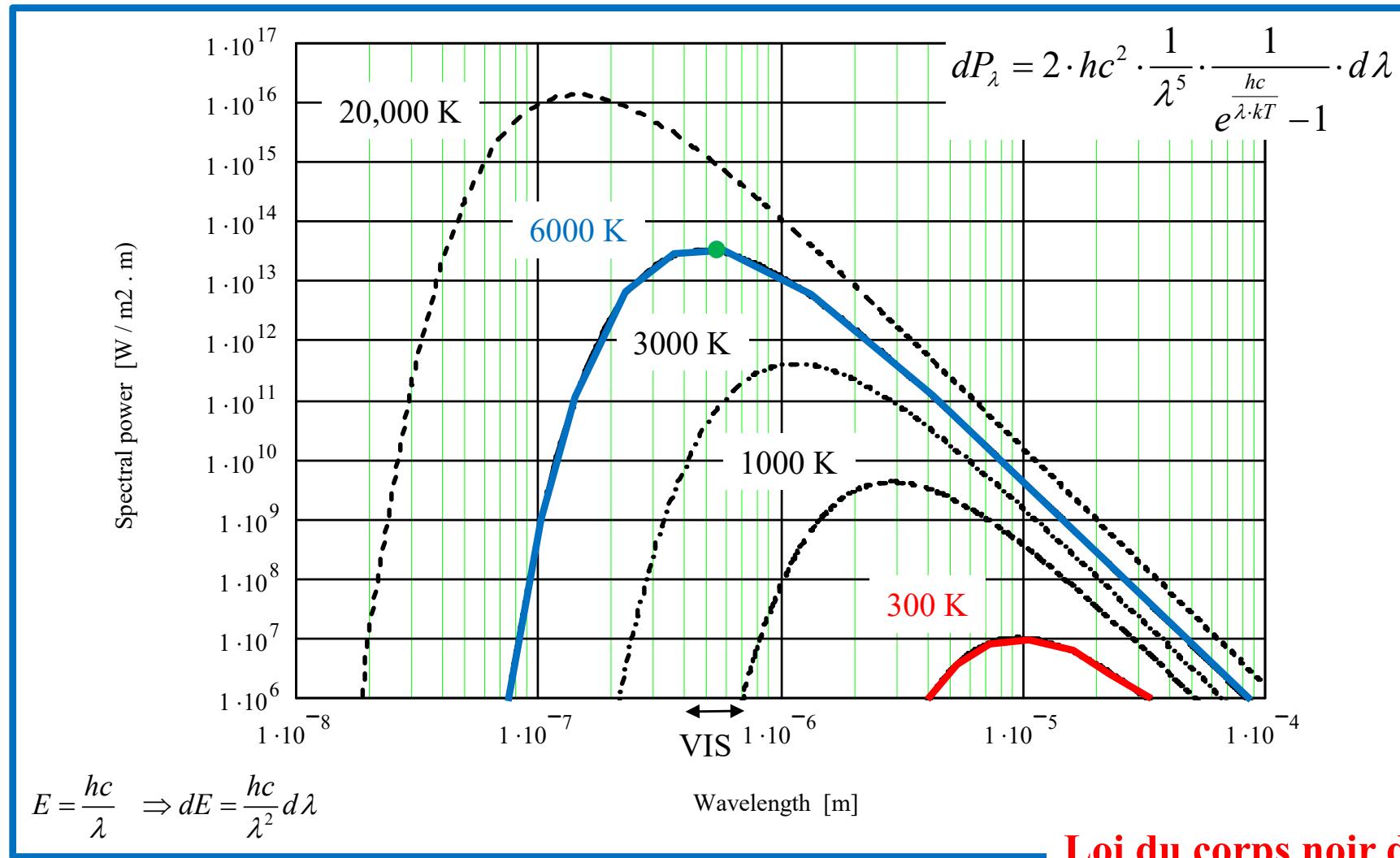
$\sigma$  = Stefan-Boltzmann constant

$$\sigma = \frac{\pi^2 k^4}{60 \cdot c^2 h^3}$$

**Loi de Stefan-Boltzmann**

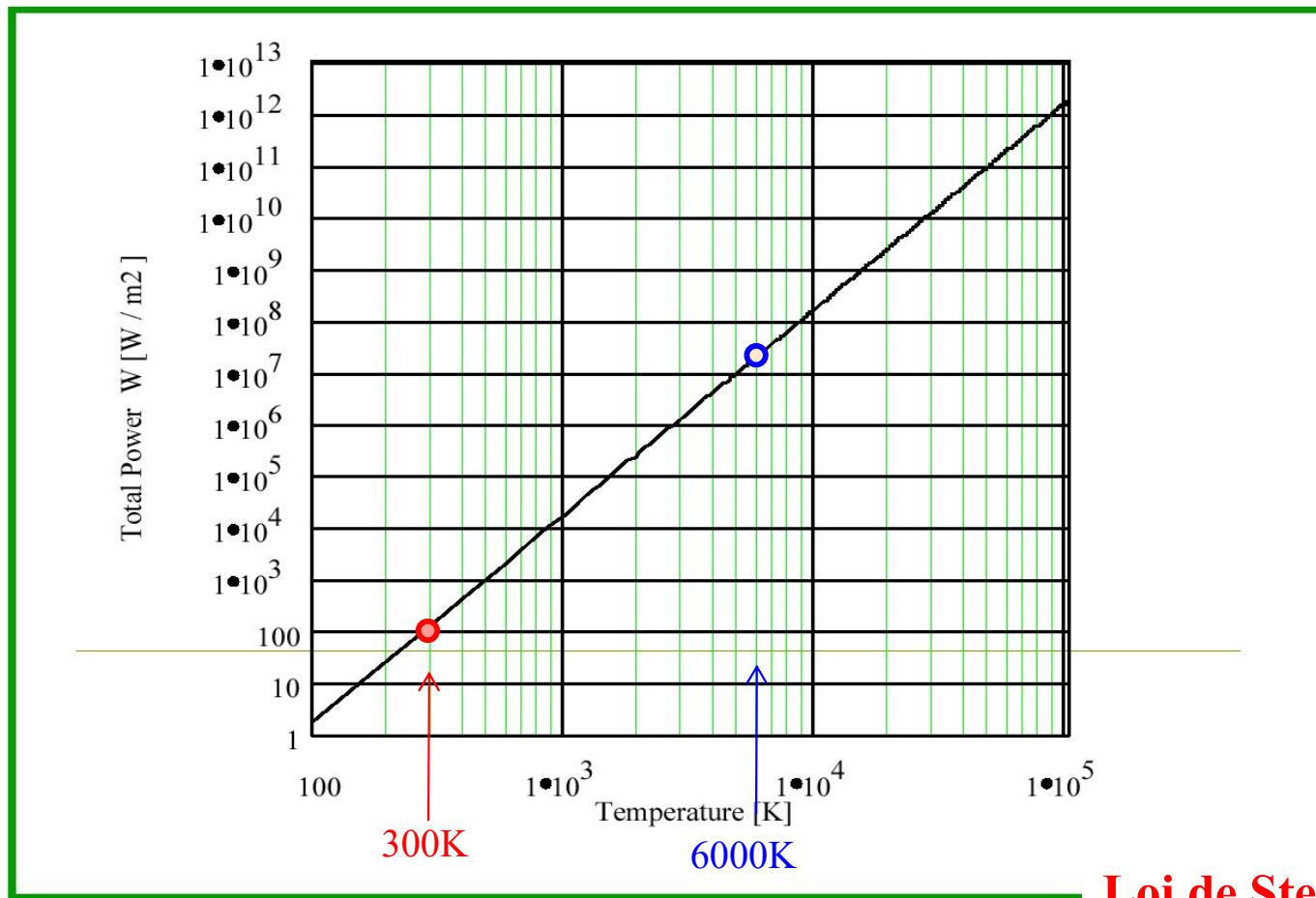
# Example: Planck's Law

## a) Black-body emissions at 300, 1000, 3000, 6000 et 20,000 K



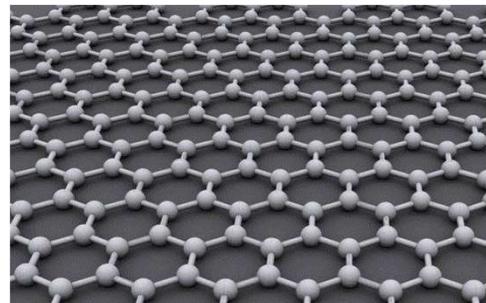
## Example: Black-body

### b) Total power emitted per solid angle and area



Loi de Stefan-Boltzmann

## Densité de porteurs dans le graphène



Densité d'états 2D:

$$\rho_K^{2D} \cdot dK \cong 2\pi K \cdot dK$$

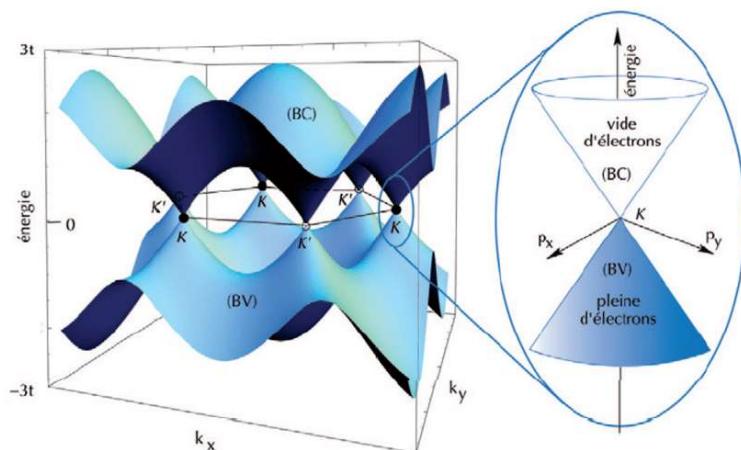
Dispersion linéaire

Similaire aux photons !!

$$E = v_g \cdot \hbar \cdot K$$



$$\rho_E^{2D} \cdot dE \cong E \cdot dE$$

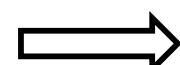


Taux d'occupation:

Principe d'exclusion  $\rightarrow$  Fermi-Dirac

$$F_{FD} = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

6 vallées      2 spins

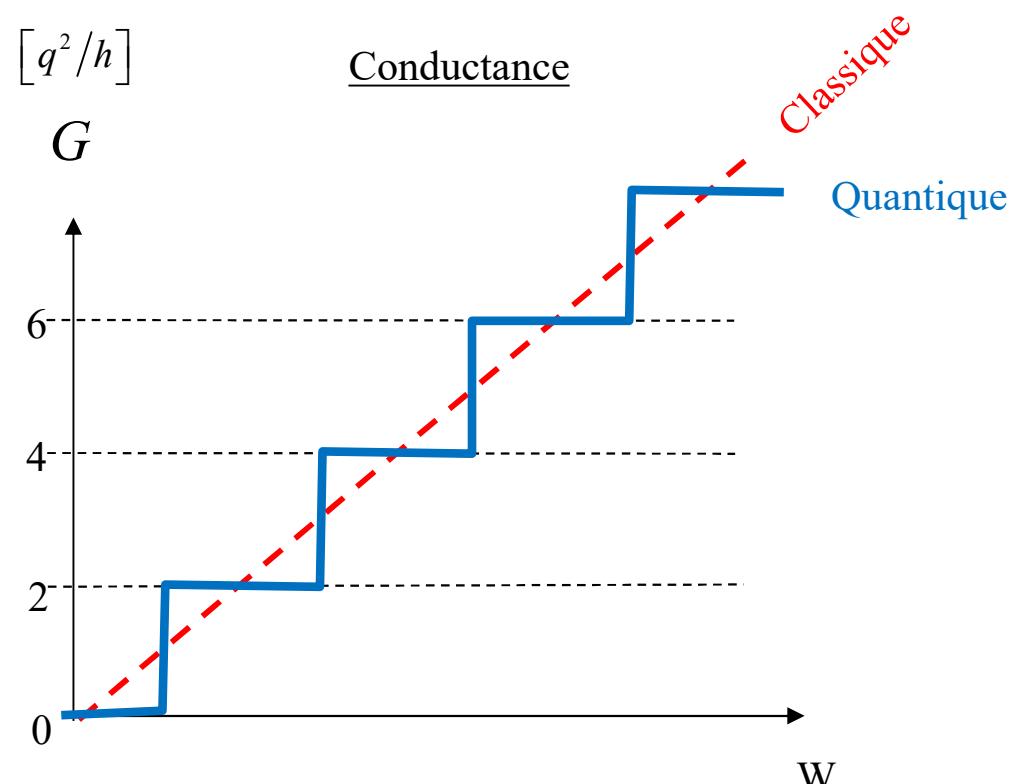
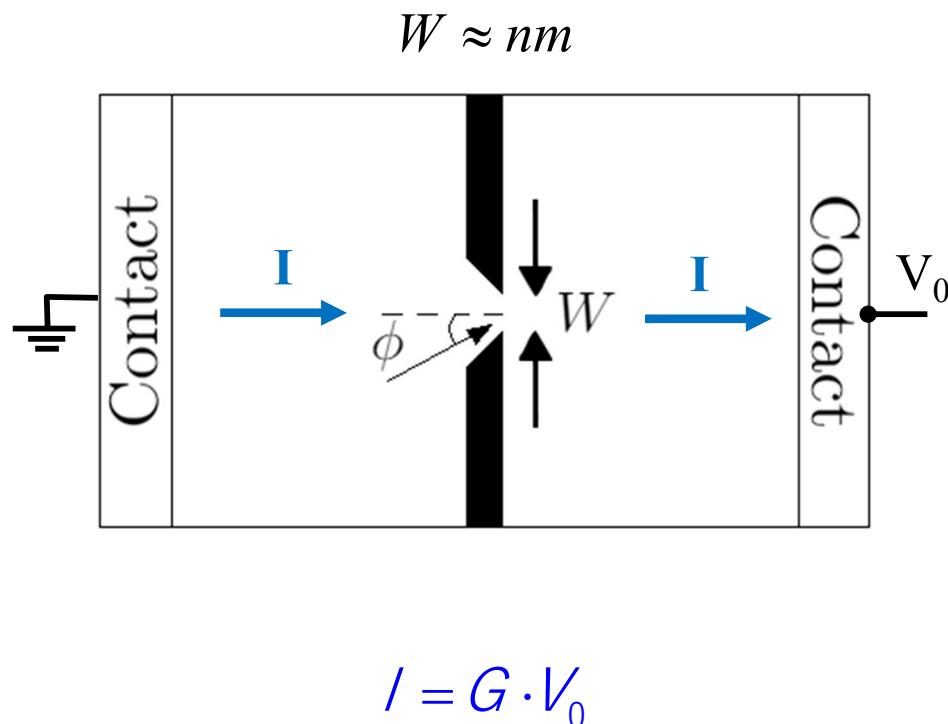


$$n = \frac{6 \cdot 2}{2\pi \cdot v_g^2 \cdot \hbar^2} \cdot \int_0^{\infty} E \cdot \frac{1}{e^{\frac{E-E_F}{kT}} + 1} \cdot dE$$

$$\left[ \frac{1}{m^2} \right]$$

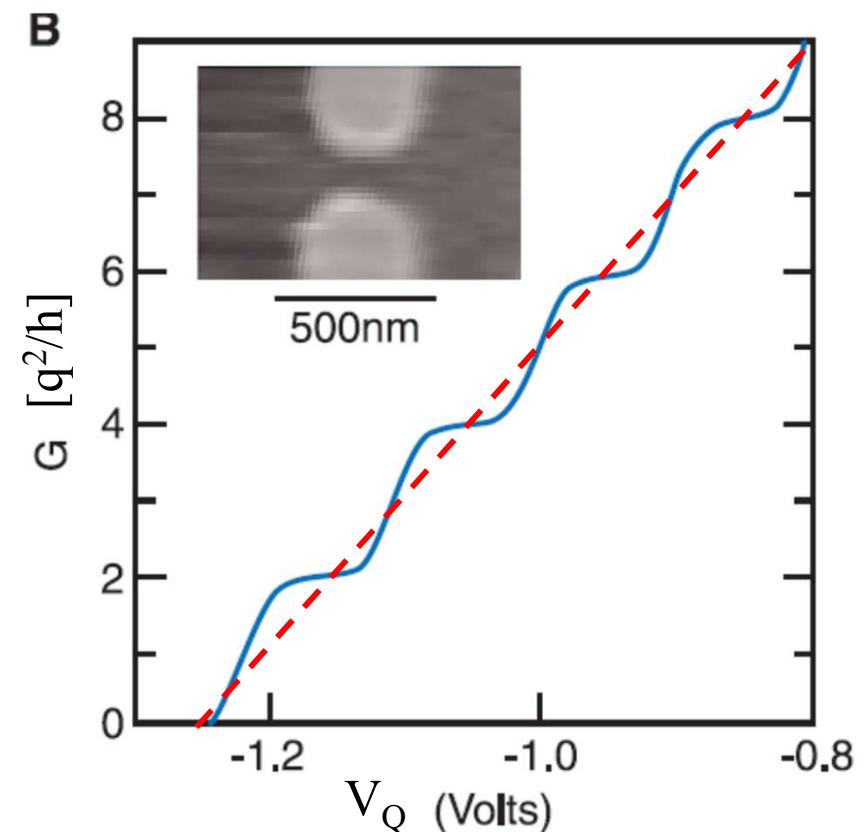
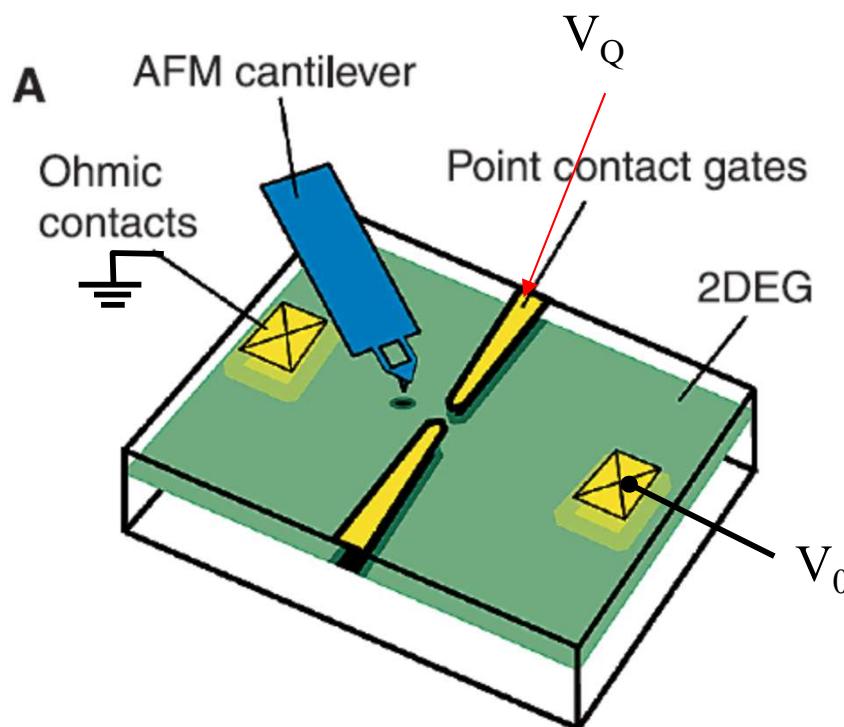
# Quantum Point Contact

# Quantum Point Contact: principle



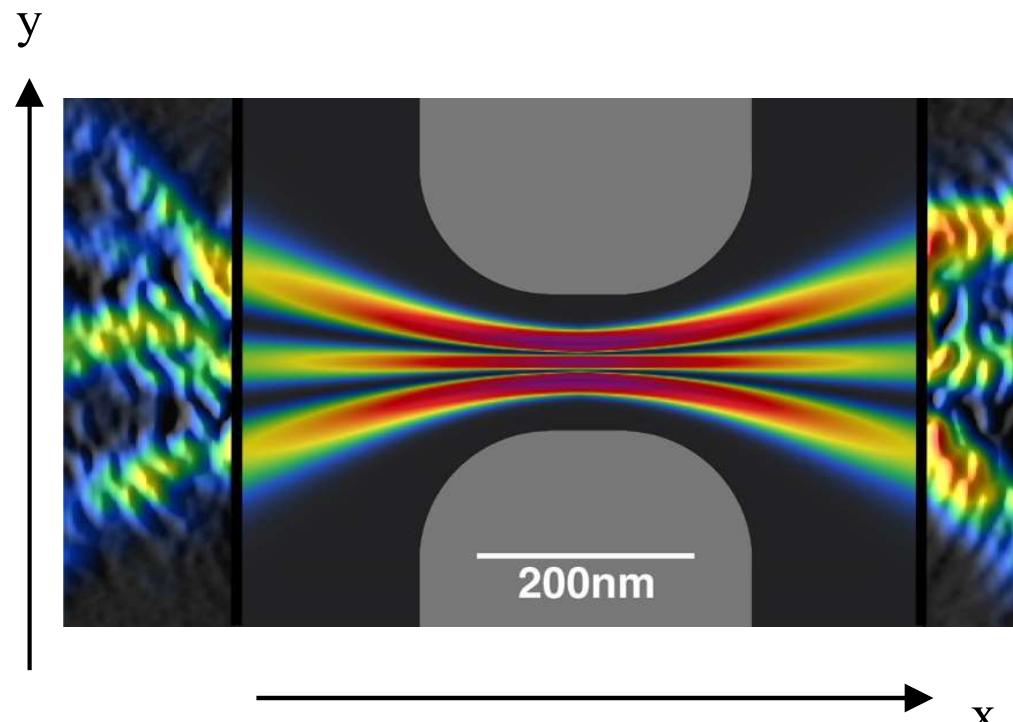
**G peut être contrôlée par W**

# Quantum Point Contact: principle

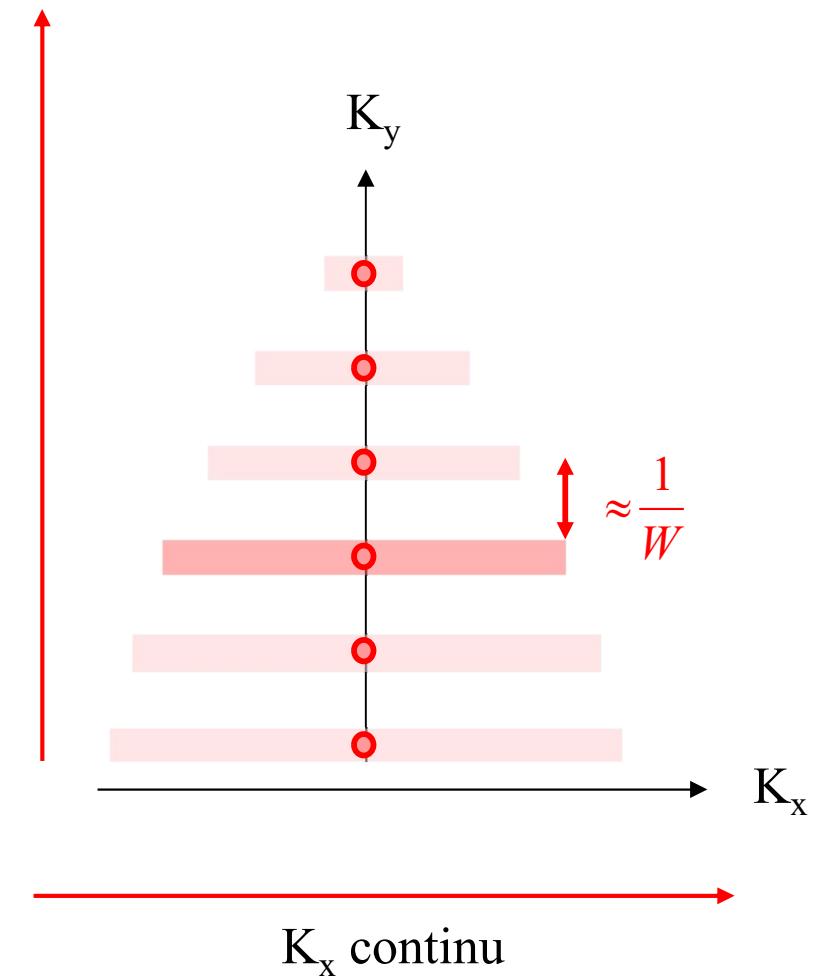


W diminue (jusqu'à disparaître) plus  $V_Q$  est négatif

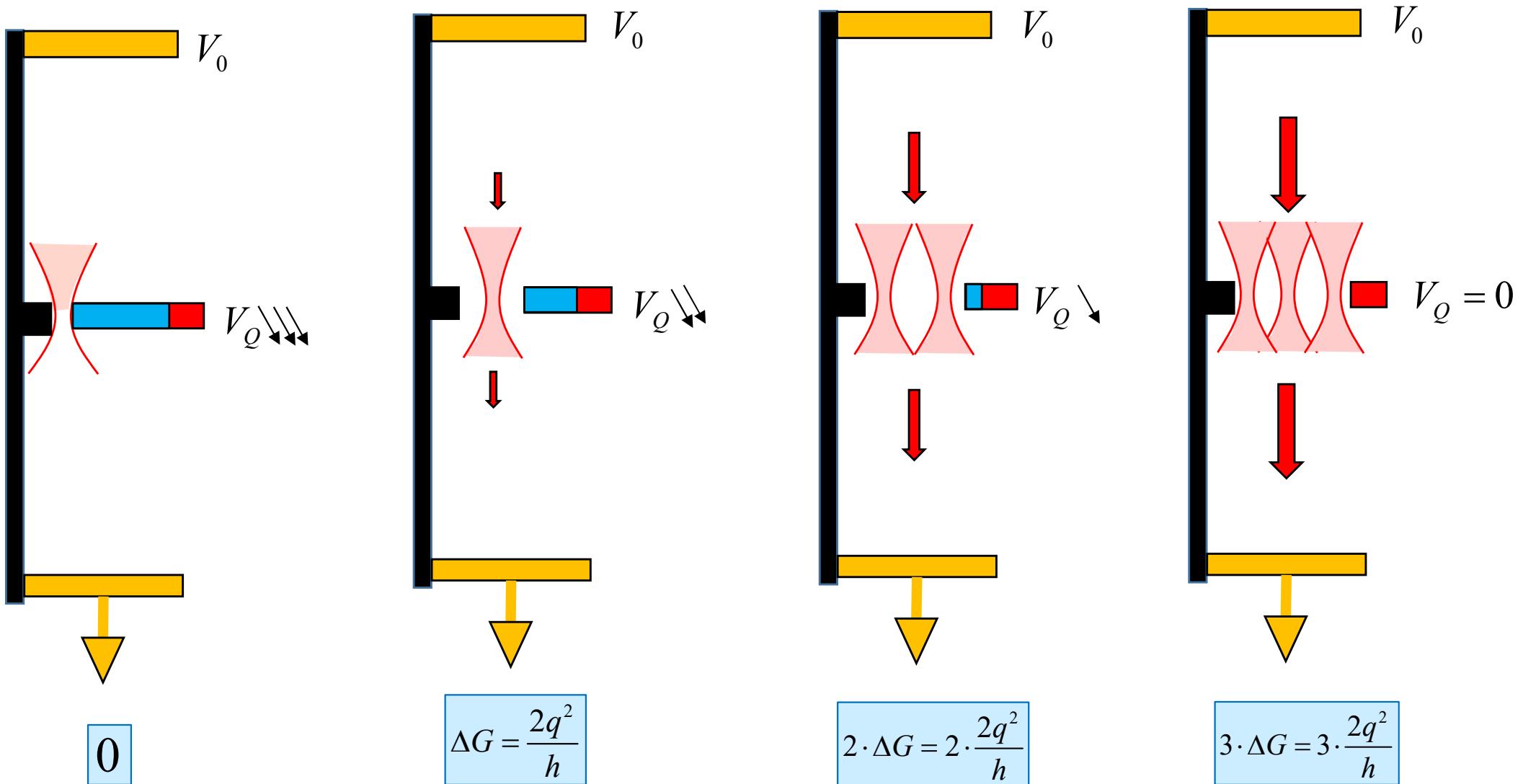
# Quantum Point Contact: principle



$K_y$  discret



# Quantum Point Contact: principle



# Conductance quantique

Densité d'états 1D  
(dans le «fil quantique 0,m»):

$$\rho_{E,e}^{1D} \cdot dE = \frac{1}{\pi} \cdot \frac{1}{\hbar} \frac{1}{\sqrt{2(E - E_{0,m}) / m^*}} \cdot dE = \frac{2}{h} \cdot \frac{1}{|v|} \cdot dE \quad \left[ \frac{1}{m} \right]$$

vitesse

Courant du  
«fil quantique 0,m»:

$$I_{0,m} = \int 2 \cdot (-q \cdot v) \cdot \frac{\rho_{E,e}^{1D}}{2} \cdot F_e \cdot dE$$

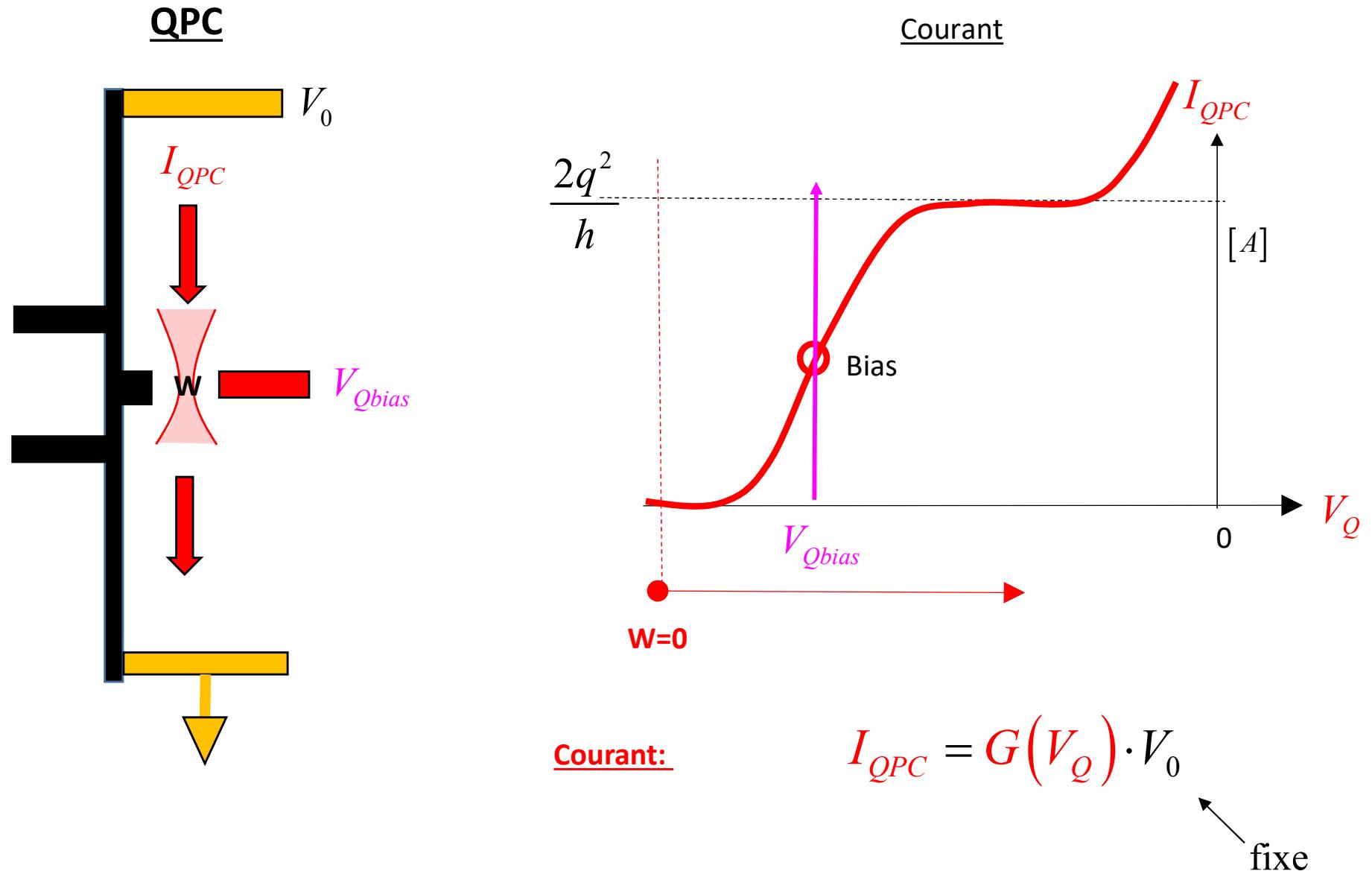
$$= -\frac{2q}{h} \cdot \Delta E = -\frac{2q^2}{h} \cdot V_0 \quad [A]$$

$$\Delta E = q \cdot V_0$$

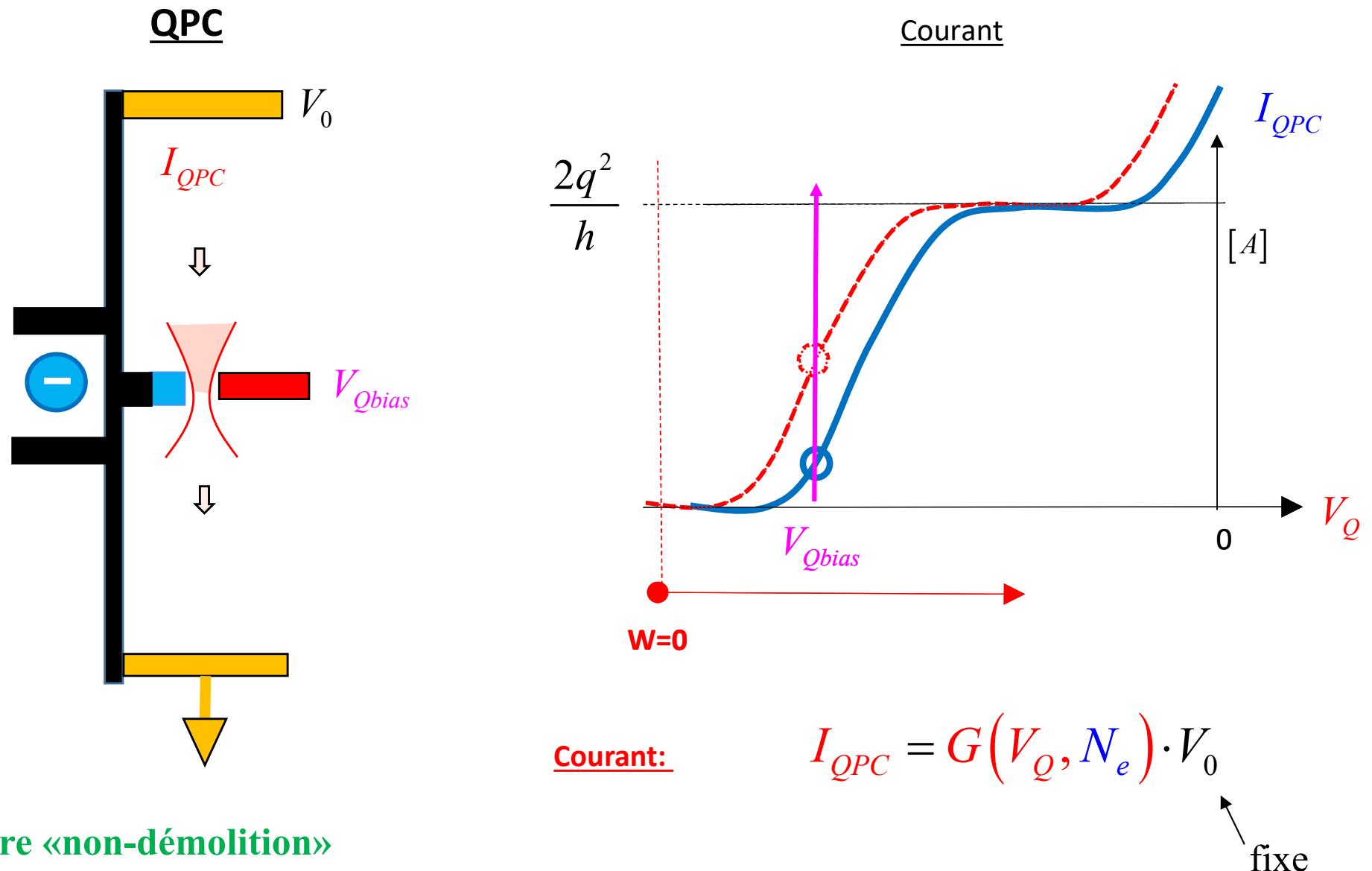
→ Conductance

$$\Delta G_{0,m} = \left| \frac{\partial I_{0,m}}{\partial V_0} \right| = \frac{2q^2}{h} \quad \left[ \frac{1}{\Omega} \right]$$

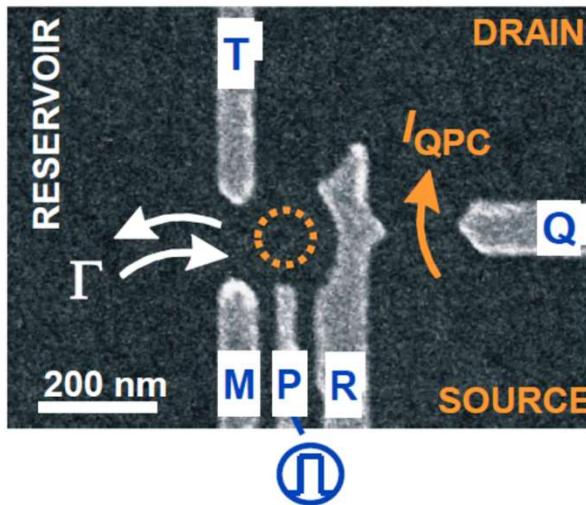
# Quantum Point Contact: principle



# Quantum Point Contact: principle



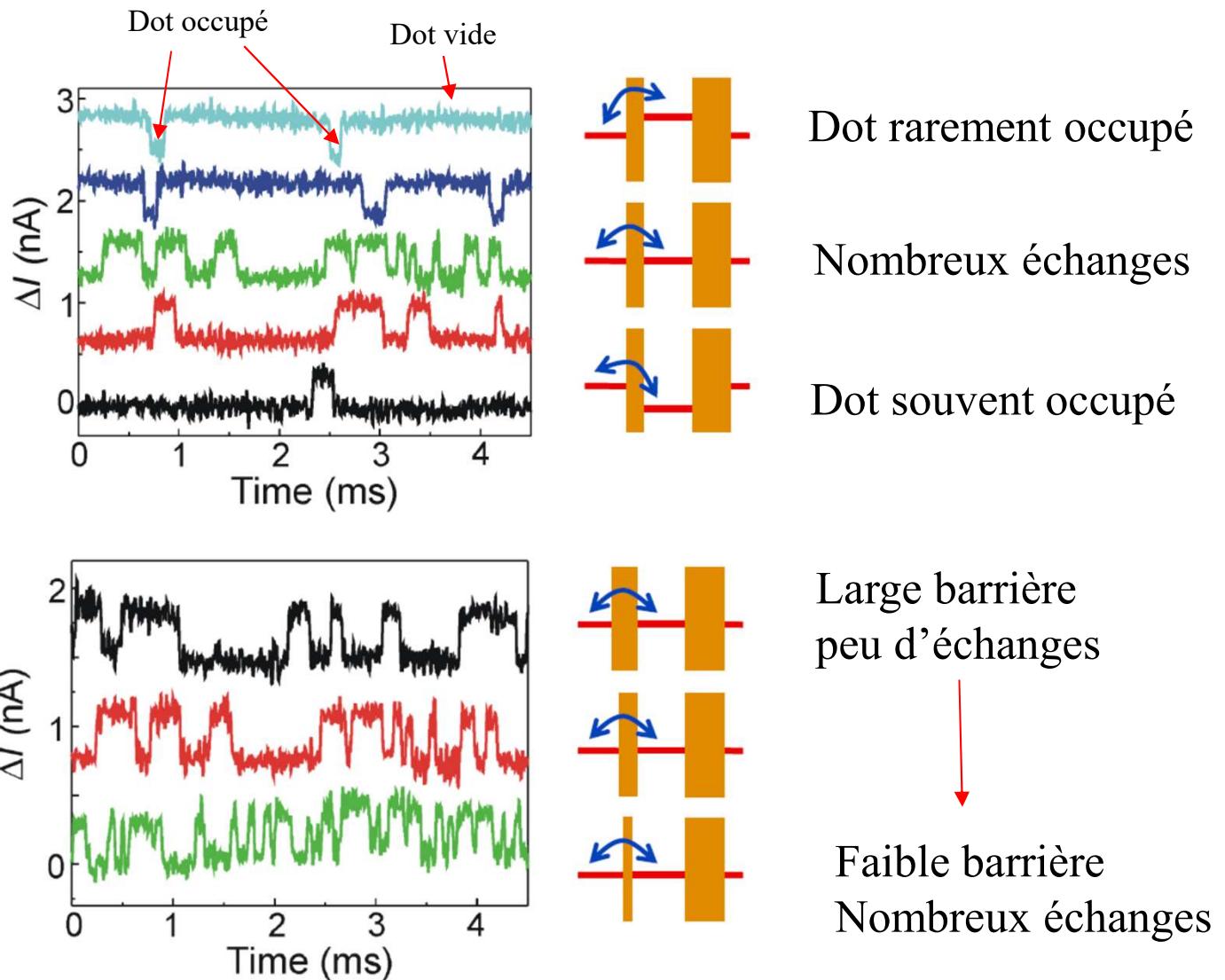
# Exemple: Spin qubit mesure par Quantum Point Contact (QPC)



- $V_{SD} = 1 \text{ mV}$
- $I_{QPC} \sim 30 \text{ nA}$
- $\Delta I_{QPC} \sim 0.3 \text{ nA}$
- Shortest steps  $\sim 8 \mu\text{s}$

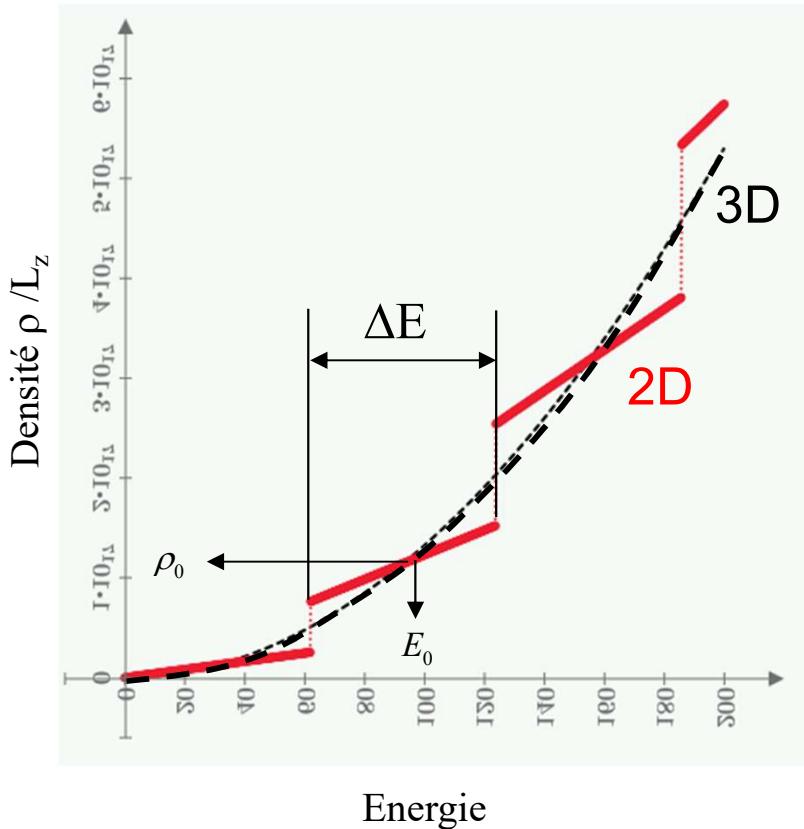
See single electrons jump on/off the dot in real-time

Vandersypen et al, APL 2004



- **Conductivité électrique**
- **Répartition des vitesses dans un gaz parfait**
- **Vibrations cristallines: phonons**
- **Capacité et conductivité thermique**
- **Pouvoir thermoélectrique, effet Peltier**
- **Effet Hall, magnétorésistance, paramagnétisme de Pauli**
- **Piezorésistance (jauge de contrainte)**
- **Bruit thermique d'une résistance**
- **Pression de Casimir**
- ...

# Pression de Casimir



Energie 2D

$$E_{2D} = \int_0^{\infty} \frac{E}{2} \cdot \rho_{2D}(E) \cdot 1 \cdot dE \rightarrow \infty$$

Energie 3D

$$E_{3D} = \int_0^{\infty} \frac{E}{2} \cdot (L_z \cdot \rho_{3D}(E)) \cdot 1 \cdot dE \rightarrow \infty$$

Energie de Casimir

$$E_{cas} = E_{2D} - E_{3D} = -\frac{\hbar\pi^2c}{720} \cdot \frac{1}{L_z^3}$$

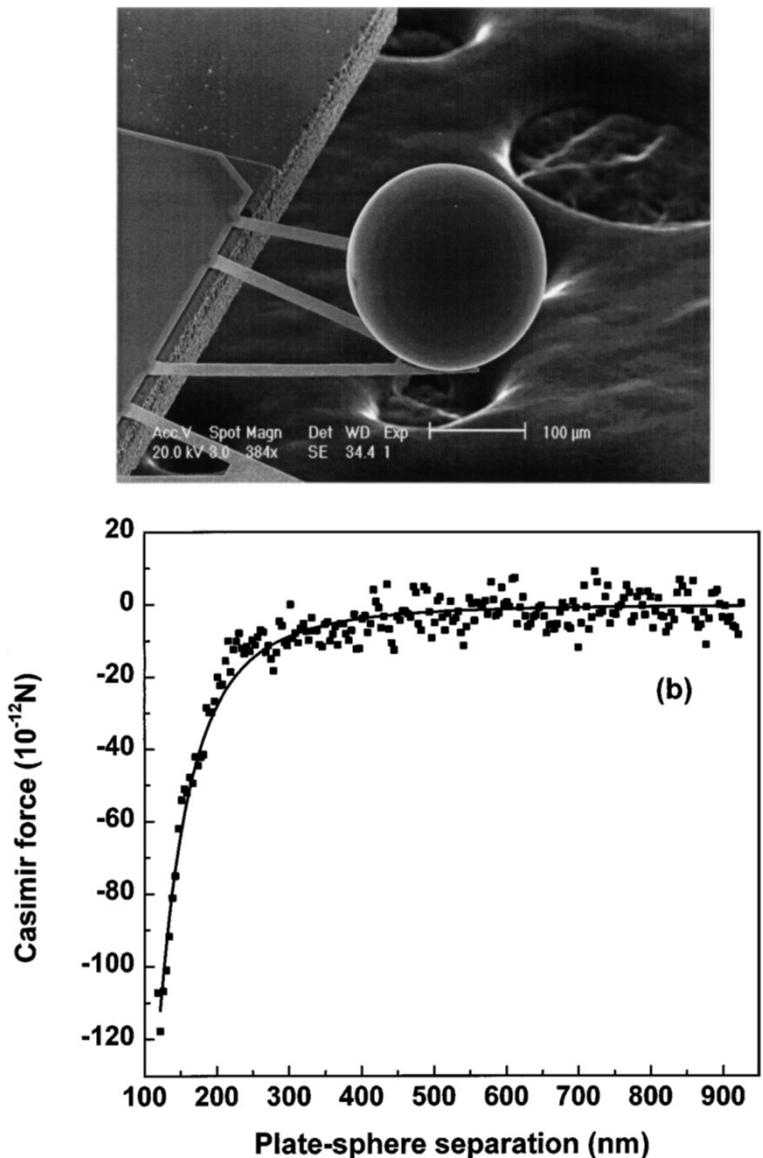
Energie du vide dans un mode d'énergie E:

$$\frac{1}{2}\hbar\omega = \frac{1}{2}E$$

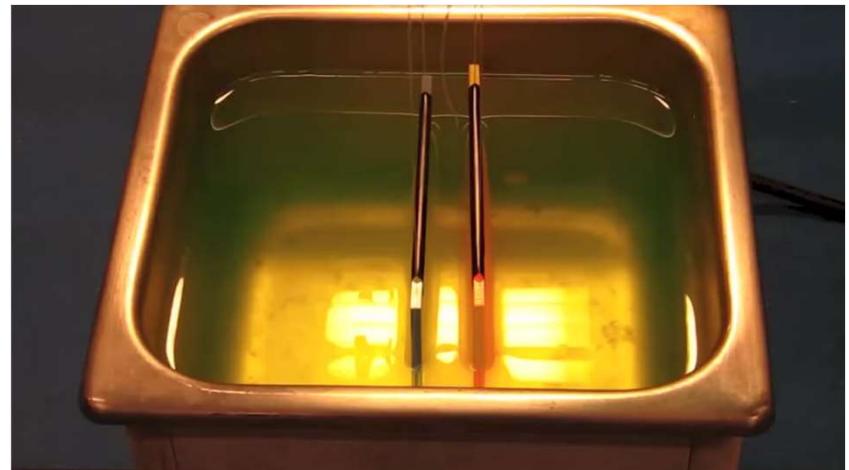
Pression de Casimir

$$P_{cas} = -\frac{\partial E_{cas}}{\partial L_z} = -\frac{\hbar\pi^2c}{240} \cdot \frac{1}{L_z^4}$$

# Pression de Casimir



## Analogie



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